Figure 1: Not Fred

Fred's misguided sports parents, and what they learned from Lance Armstrong

Edward: draft, Nov 13, 2014

(Note that this lecture contains a bunch of basic algebra. When you first work through these notes you might want to concentrate on the graphs and slide over the math. On the second go, try and understand some of the math; it is only basic algebra.)

Fred is a cross-country skier who races on the Boulder Nordic Junior Race Team, BNJRT.

Fred however does not like to train (ski for miles to prepare for races). he just wants to "play around"—what do you expect from a eight-year-old kid?

his parents, both bad sports parents (Boulder is full of them), has come up with an economic plan to get him to train more (produce more miles).

They will pay him $\$1/mile$ for every mile he skis on Saturdays. his parents will provide him with all needed equipment and drive him to the cross-country trails at Eldora.

All Fred has to do is provide the time and effort to crank out the miles. If he skis enough, he can buy a new cell phone, download more songs, whatever.

The problem for Fred is how much of his Saturday should he allocate to the production of ski miles. It is his decision, and it is a production decision.

Fred is a firm producing a product (ski miles) which he can sell for \$1 per unit. his problem is determining how many miles to ski, how many units of output to produce.

Fred is a competitive firm that can produce ski miles and sell them at a \$1 per mile. For example, Fred's problem is the same problem a wheat farmer faces–how many bushels of wheat to produce give the world world price of wheat.

Put simply, I am presenting a theory to explain the behavior of a competitive firm, the theory as described in KW. I am simply doing it using Fred as an example of a competitive firm.

Keep in mind that you produce many activities. For example, next week you will produce a score on the secon midterm. Tonight, you or someone you know will produce the dinner you eat.

1 Fred produces miles skied, m, using two inputs, ski equipment, s, measured in quality units, and his labor/time, l, measured in hours.

The **maximum** number of miles he can ski given the amount of equipment he has, s, and the amount of labor/time, l, he commits to skiing can be described by something economists call a production function, a mathematical function,

$$m = m(s, l)$$

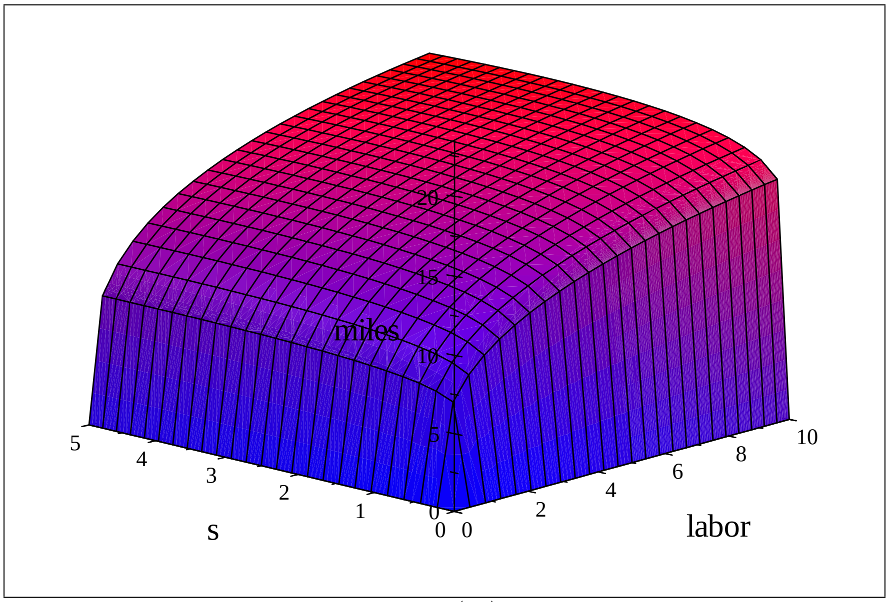
where m is the maximum number of miles he can ski with skiing equipment level s, and labor hours, l. Read this as the maximum number of miles depends on the quality of his equipment and the number of hours he works skiing.

In cross country skiing, s reflects the quality of the skis, boots and poles, and the wax and wax job.

This if Fred's shortrun *production function.* It identifies **maximum** output as a function of the level of inputs.

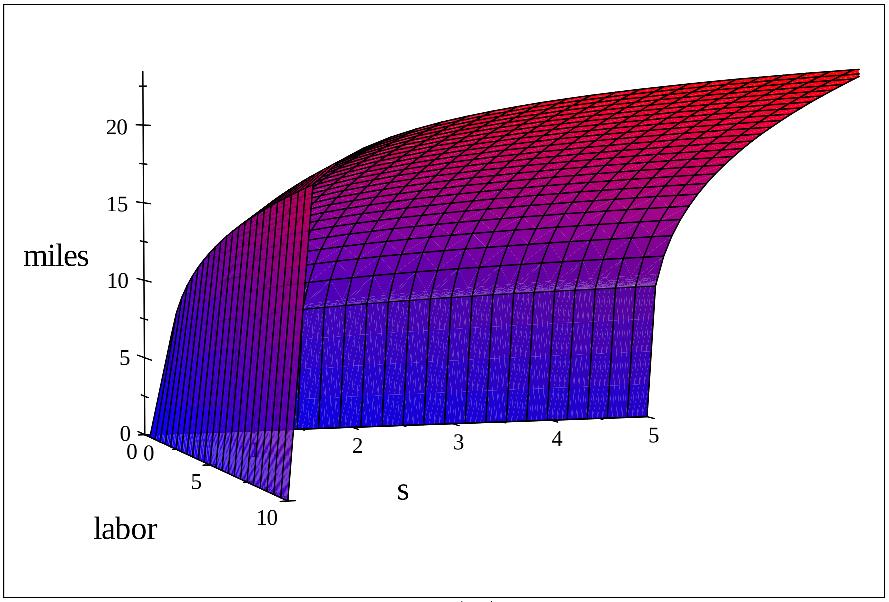
The form of his production function will depend on his body type, flexibility, strength, endurance, skill set, and his threshold for pain. Also the trail conditions

For example,



m=m(s,l)

The same function from a different angle



m=m(s,l)

Note that if either $s = 0$ or $l = 0$, miles are zero. If one skis for zero time one goes zero miles, and if one does not have skis on, one is not skiing.

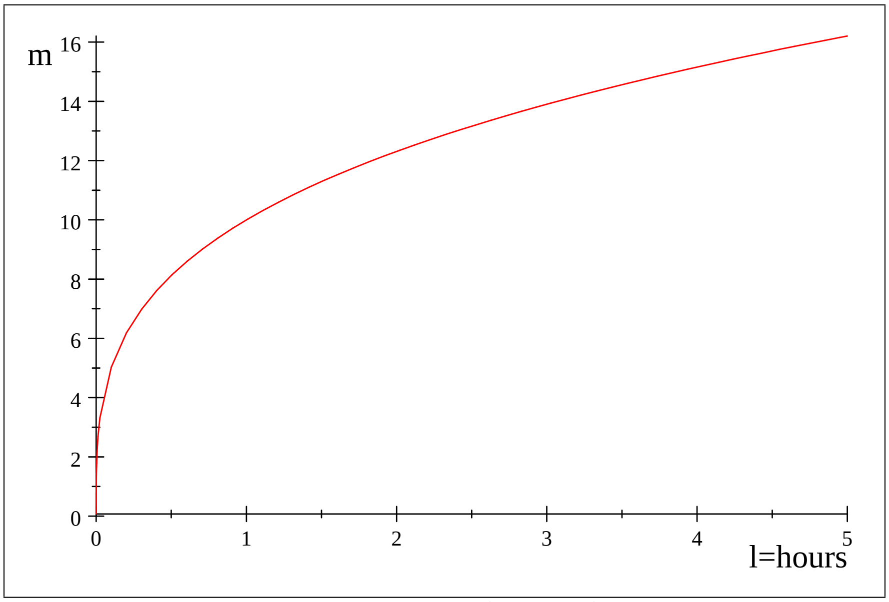
Assume it is the shortrun and Fred's equipment for the season has already been purchased, so s is fixed at one unit $s = \overline{s} = 1$. In this case Fred's production function for the season is[1]

$$m = m(s, l) = m(\overline{s}, l) = m(1, l) = g(l)$$

For my example, I am going to assume $m = g(l) = 10(l)^{.3})^2$. Graphing this one-input production function.

[1] Note how I gave the function a new name, "g" when I made m a function of only l.

[2] I simply chose a mathematical function that would give me the shape and dimentsions I thought might reasonalbly approximate the production of ski miles.



Fred's shortrun production function for ski miles

As expected, if $l = 0$, $m = 0$ – this follows from the laws of physics (you can't produce something from nothing)

Note that Fred can operate on the red line but also at any point between the red line and the horizontal axis. (This area identifies his *production set* consists of all the technically feasible combinations of the input and the output.). It would be foolish for Fred to choose an input output combination that was not on the red line.

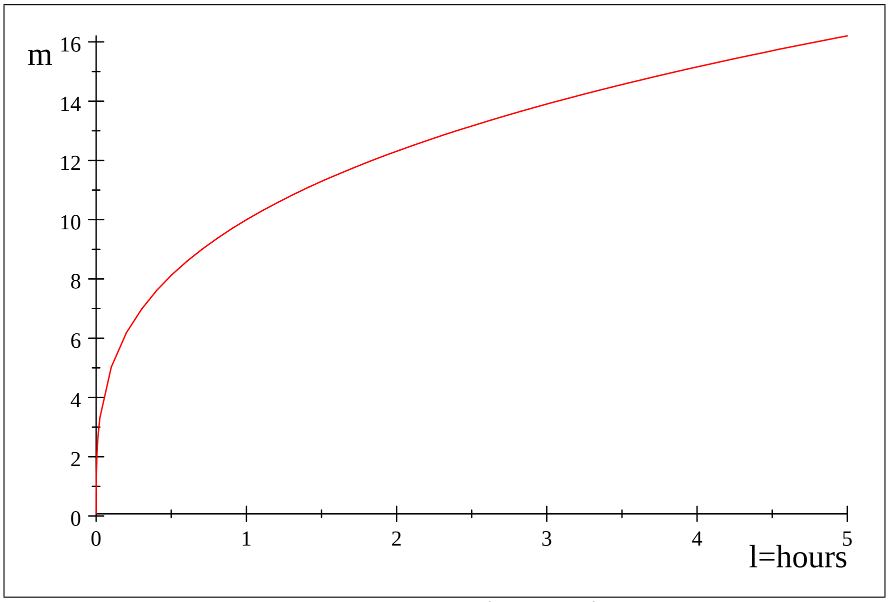
Given the production function I specified, If he skis for two and a half hours he will go $g(2.5) = 10(2.5)^{.3} = 13.2$ miles.

And if he skis five hours he will ski $10(5)^{.3} = 16.2$. Fred starts off fast and then quickly gets slower and slower: this is the only way he knows to ski (his state of technical knowledge for producing ski miles).

His productivity declines as the number of hours he skis increases. We might measure his productivity on the margin as how many more miles he skis when he skis for one more hour. This is called his *marginal productivity of labor*, MP_l, in the production of ski miles.

How *productive* is Fred on the *margin;* that is, what is his *marginal product of labor?*

Looking at the graph of the shortrun production function we see that



Fred's shortrun production function for ski miles

In the first hour he covers 10 miles

- $10(1)^{.3} = 10.0$
- $10(2)^{.3} = 12.3$ and $12.3 - 10 = 2.3$, so 2.3 miles in his second hour.
- $10(3)^{.3} = 13.9$ and $13.9 - 12.3 = 1.6$ miles in his third hour
- $10(4)^{.3} = 15.2$ and $15.2 - 13.9 = 1.3$ miles in his fourth hour.
- $10(5)^{.3} = 16.2$ and $16.2 - 15.2 = 1$ mile in his fifth hour

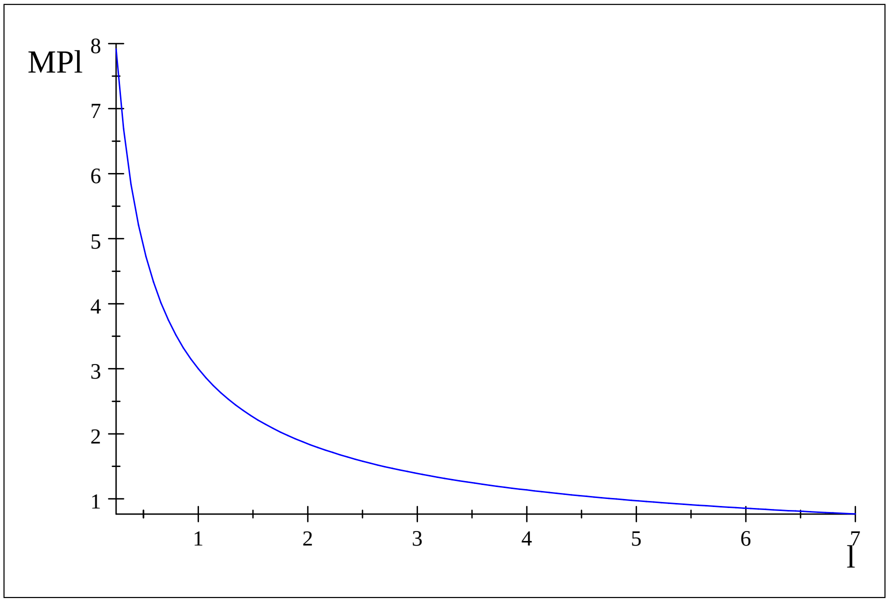
Fred's output	first hour	second hour	third hour	fourth hour	fifth hour
total miles	10	12.3	13.9	15.2	16.2
additional miles	10	2.3	1.6	1.3	1

His speed in the fifth hour is 1/10 of his speed in the first hour - after four hours of skiing, he is really dragging.

Graphing Fred's marginal product of labor

$MP_l(l) = \frac{\triangle m}{\triangle l} = \frac{dg(l)}{dl} = .333(l)^{-.73}$

Note that $MP_l(l)$ has a negative exponent, so declines as l increases

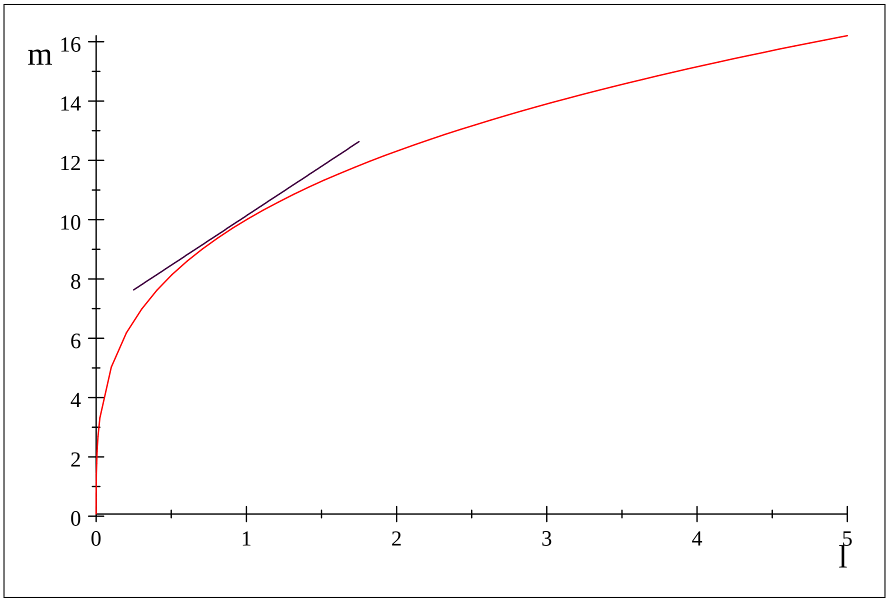


Fred's marginal product of labor

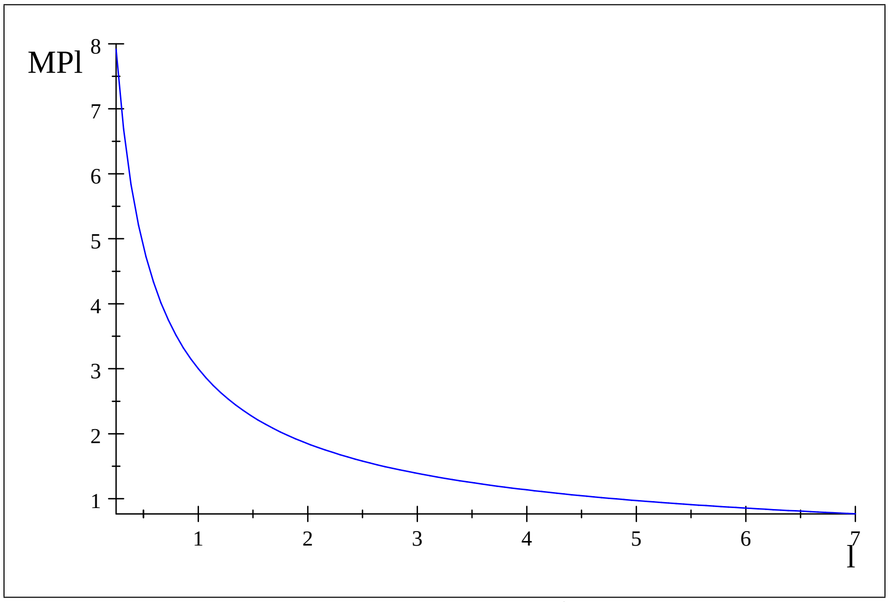
As the numbers in the table indicate, and the graph shows, his marginal product of labor in the production of ski miles continuously declines, first rapidly, then more slowly, but it is never negative (he never stops and never goes backwards, but he is eventually crawling along the trail).

[3] The mathematically inclined might note that the marginal product of labor is the derivative of the production function with respect to labor. That is, how much miles increase when ski time is increased a bit.

Fred's marginal product of labor when $l = l_0$, $MP_l(l_0)$, is the slope of his production function at $l = l_0$. It is a measure of how much his miles will increase if he skis one more hour than l_0 hours.



Fred's shortrun production function for ski miles



Fred's marginal product of labor

2 Fred's problem with skiing is that there are lots of other things to do besides skiing: texting, Facebooking, reading, hanging with friends, etc.

Because of these other options, Fred values his time at \$3 an hour. (think of this as his "wage rate", an hour of skiing costs him \$3 of fun)

Can we use his production function for ski miles and the fact that he values his time at \$3 an hour to figure out his **cost function** for ski miles–what it cost him to produce m ski miles? The cost function will identify the **minimum** cost of skiing m miles

Note that his only cost is the opportunity cost of his time. He did not have to pay for the ski equipment

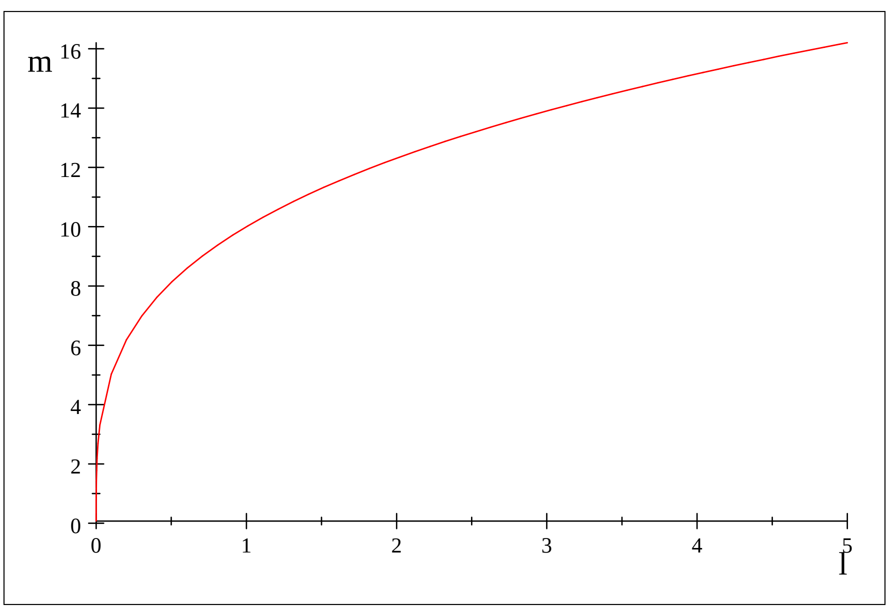
YES, we can determine his costs to produce ski miles

His *cost function* for producing ski miles is

$$c = c = (m, w) = c(m, 3)$$

where m is the number of ski miles he produces and c is the cost, in dollars, of his producing those m miles given that his "wage" rate is \$3/hr (the opportunity cost of his time)

and his production function



Fred's shortrun production function for ski miles

The cost function will depend on w in per-unit cost of an input and the technology for producing ski miles (represented by the production function.

As assumed, Fred does not have to pay for his ski equipment.

Let's see if we can figure out some points on the cost function.
What does it cost him to produce $m = 0$? If $m = 0$, $c = 0$ - it takes no time to ski zero miles.

From above we know that if he works one hour he will produce 10 miles, so if $m = 10$, $c = \$3$.

And if $l = 2$ then $m = 12.311$, and $c = \$6$

I figured out cost function (don't sweat how I did it).[4]

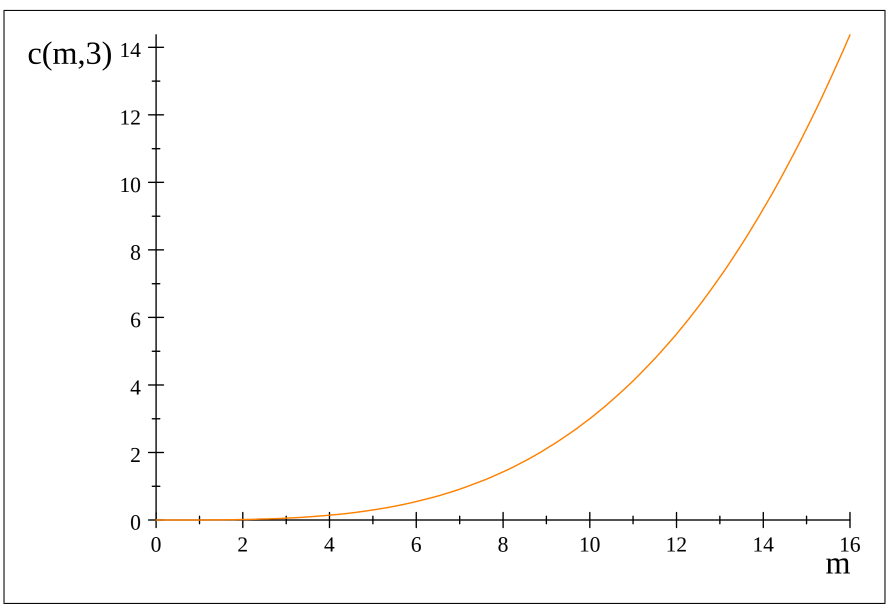
Fred's cost function for ski miles is
$c = c(m, w) = w(4.6416 \times 10^{-4})m^{\frac{10}{3}}$

For example, if $m = 10$ and $w = 1$,

$c = c(10, 1) = 1(4.6416 \times 10^{-4})10^{\frac{10}{3}} = 1.0$, recollect that Fred could ski 10 miles in one hour.

And if $w = 3$, $(4.6416 \times 10^{-4})m^{\frac{10}{3}}3 = .0013925 \times 10^{-3}m^{\frac{10}{3}}$

$c = c(m, 3) = .0013925m^{\frac{10}{3}}$



Fred's cost function ski miles if w=3

[4] The production function is $10l^{.3}$. Derive the cost function by minimizing wl subject to $m = 10l^{.3}$. Solve $m = 10l^{.3}$, Solution is: $\left\{\frac{1}{10000}10^{\frac{2}{3}}m^{\frac{10}{3}}\right\}$ which is how much labor one needs to product m, so the cost function is $\frac{w}{10000}10^{\frac{2}{3}}m^{\frac{10}{3}} = (4.6416 \times 10^{-4})m^{\frac{10}{3}}w$

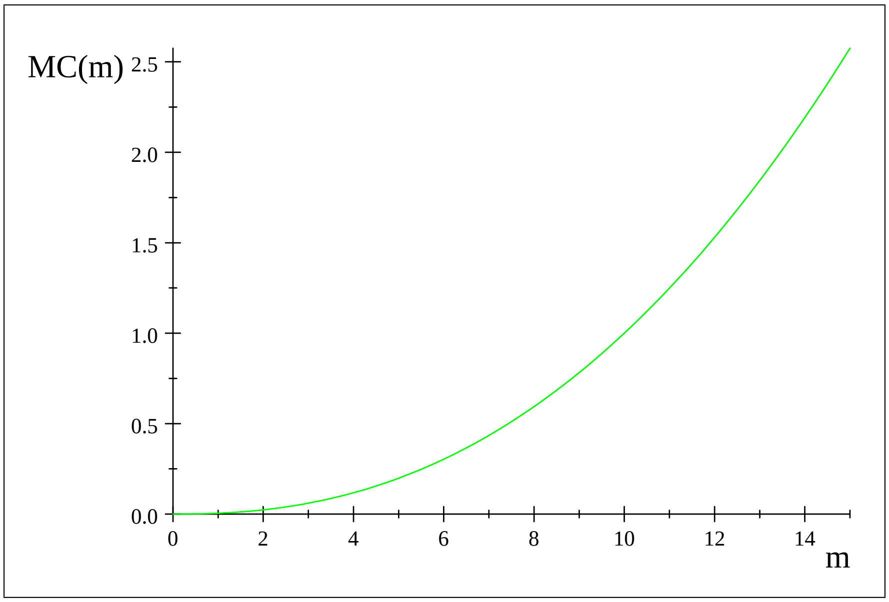
Note that in this example cost increases at an increasing rate.

Draw a production function on a piece of paper then flip over the graph and look at it from the other side. You will see the cost function associated with wage= $\$1/hr$

What does it cost Fred to produce each additional mile? It depends on how many miles he has already skied.

Fred's marginal cost function, the cost of producing each additional mile, is[5]

$$\frac{\triangle c(m,3)}{\triangle m} = \frac{d(3((.1m)^{\frac{10}{3}}))}{dm} = MC_m(m) = .004\,641\,6m^{\frac{7}{3}}$$



Fred's marginal cost function for ski miles when w=3

Marginal cost is the cost of each additional mile. For example, the marginal cost of producing the fourth mile, is the total cost of producing four miles minus the total cost of producing three miles.

[5] I am obtaining the marginal cost function by taking the derivative of the cost function with respect to miles.

For example, the marginal cost of the first mile is
$3(\,(.1)^{\frac{10}{3}}\,) - 3(\,(0)^{\frac{10}{3}}\,) = \$.001\,392\,5$ Fred skis the first mile very fast

For the second mile it is
$3(\,(.1(2))^{\frac{10}{3}}\,) - 3(\,(.1(1))^{\frac{10}{3}}\,) = \$.01\,264\,3$

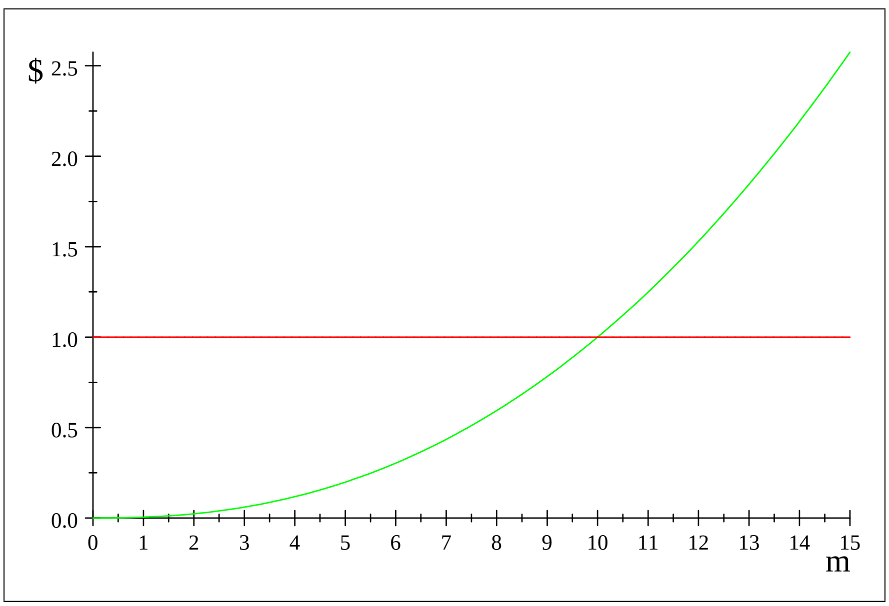
And for the tenth mile it is
$3(\,(.1(10))^{\frac{10}{3}}\,) - 3(\,(.1(9))^{\frac{10}{3}}\,) = \$0.888\,47$

3 So, how many hours should Fred ski on a Saturday?

That is, how many units of output does a competitive firm want to produce to maximize its profits.

Fred wants to spend more time skiing (producing miles) as long as he makes a "profit" on each additional minute skied.

Said another way, Fred want to keep skiing additional miles as long as the cost for each additional mile , $MC_m(m)$, is less than what he is paid per mile (e.g. $1), the constant price at which he can sell his output. In terms of the graphs, it looks like



Fred's marginal benefit and margl cost functions for miles.

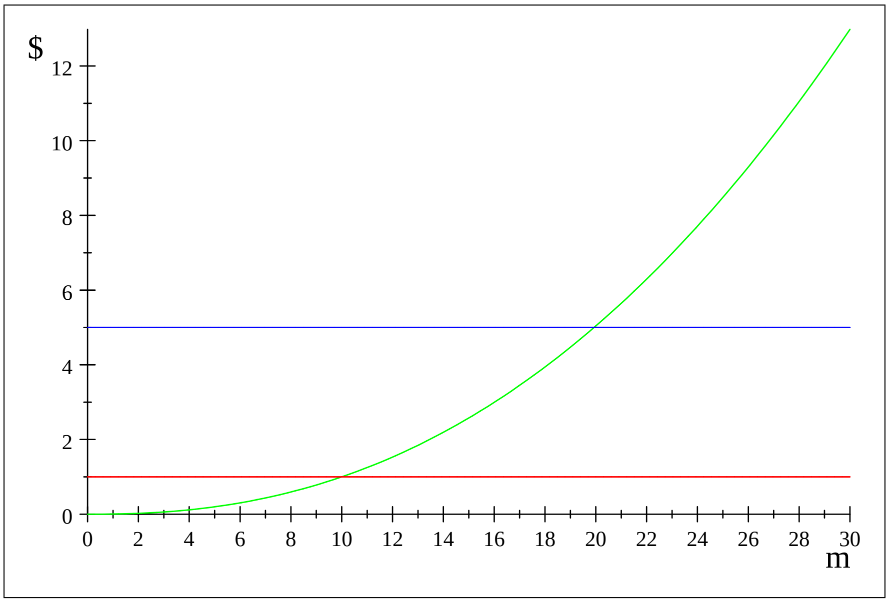
The red line is at $1 his revenue per mile (what his parents pay his for each mile skied). Since price is constant at $1 a mile, Fred's total revenue (money he gets from selling miles) increases by $1 for every additional mile he skiis; it is his marginal revenue, $MR = p_m$.[6]

[6] This is always true for a competitive firm. For the cometitive firm price is exogenous.

Fred want to ski approx. 10 miles; any more makes him worse off. his profits are maximized at the point where price equals marginal cost $p_m = \$1 = MC(m)$

If his parents want him to ski more his parents have to increase the amount they pay him per mile

How much would he choose to ski if his parents paid him \$5 mile? Looks like around 20 miles



Fred's marginal benefit and marg cost functions for miles.

When he is being paid \$1 mile, his profits are maximized at an output level of 10 miles, at \$5 a mile his profits are maximized at approx. 20 miles.

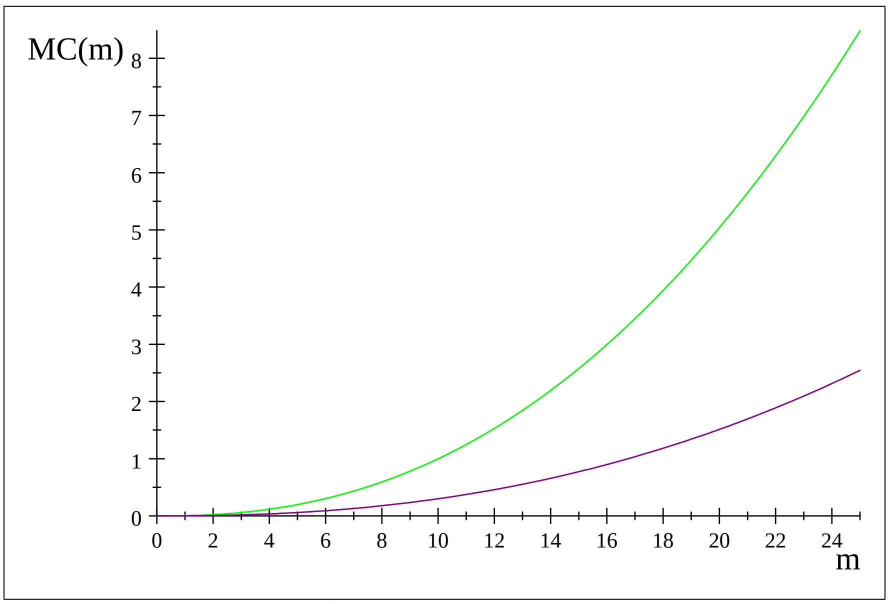
If his parents want him to ski more miles without increasing what his parents pay per mile, his parents will have to pour Red Bull down his throat, a story for later.

Fred want to produce miles up to the point where price equals marginal cost.

3.1 *Now that we have figured out what Fred should do, let's divert for a moment and look at Fred's costs in another way.*

Let's look at Fred's average cost function. Average cost, $AC(m,3) = \frac{c(m,3)}{m} = \frac{.001\,392\,5m^{\frac{10}{3}}}{m} = .001\,392\,5m^{\frac{7}{3}}$, is total cost divided by the number of miles produced.

Let's graph Fred's average cost function for ski miles on the same graph as his marginal cost function for ski miles



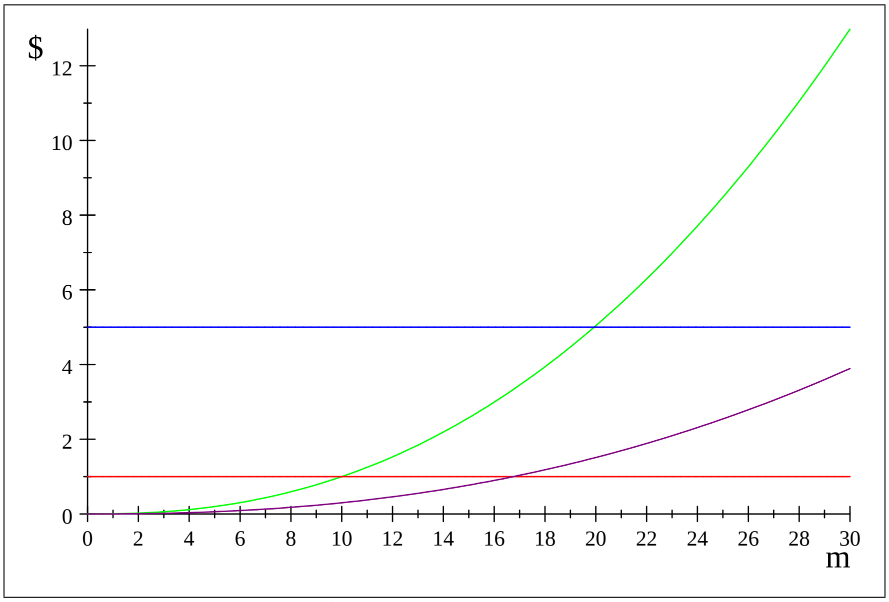
Marginal cost function green, average cost purple

Note that Fred's marginal cost function lies above his average cost function.

If marginal cost is above average cost it pulls average cost up. If, alternatively, marginal cost is below average cost it pulls average cost down.[7] Make sure you understand the distinction between marginal cost and average cost.

Let's look at Fred's marginal revenue, what his parents pay him per hour, and the marginal cost function for ski miles, but now with the average cost function added.

[7] As long as marginal cost is above average cost, even if it is decreasing, it is pulling average cost up.

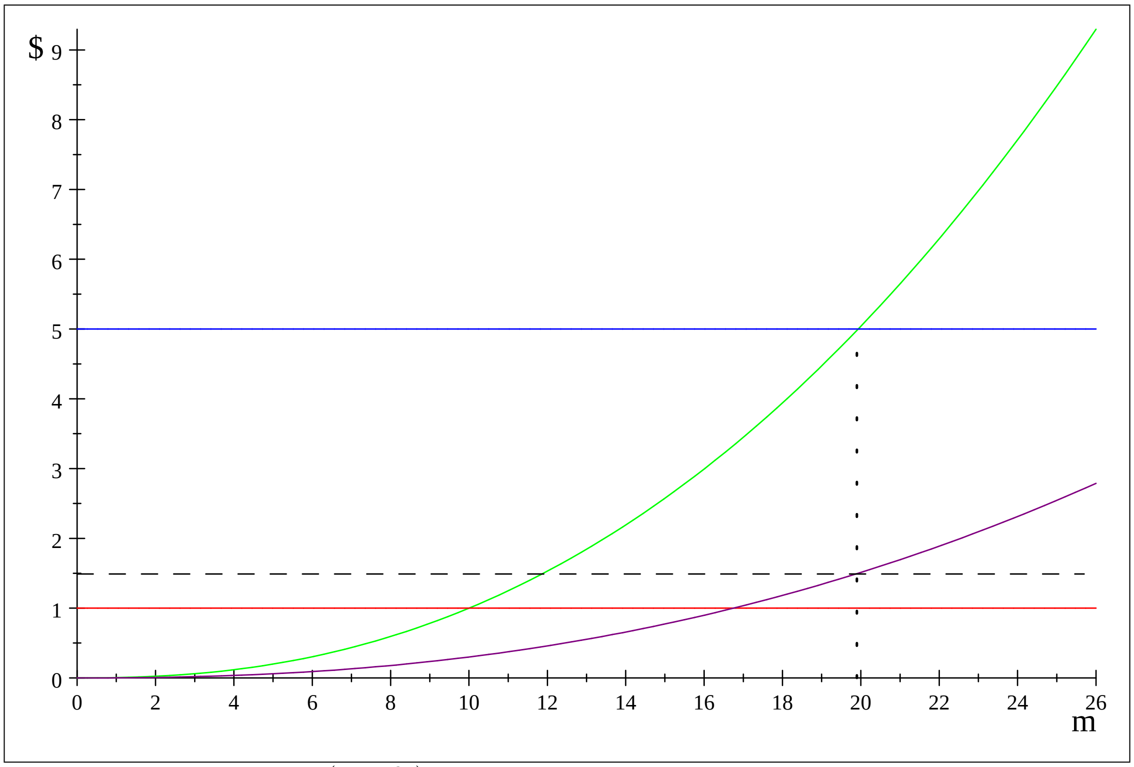


marginal benefit,marginal cost, and average cost.

The red line corresponds to $\$1/hr$ and the blue line $\$5/hr$

Note that at the point where marginal benefits/revenue equals marginal costs, average cost is less than marginal cost.

3.2 The net benefits/profits to Fred can be represented as in a number of different ways on this graph. Show me two different area representations of Fred's maximum profits from the production of ski miles.



marginal revenue (benefit),marginal cost, and average cost.

Two possible prices are represented on the graph: \$5 and \$1
The black dashed line identifies average cost when $m = 20$

If his parents pay him $\$5/mile$, Fred's profits are the rectangle bounded by the blue line at the top, the black dashed line at the bottom, and the vertical dotted black line on the right.

But, this rectangle equals the area above the green line and below the blue line between zero and 20 miles.

If his parents pay him $\$1/mile$?

Mathematically: If his parents pay Fred $\$1/mile$, he wants to find the m such that $1 = MC(m)$; that is, where price equals marginal cost. He will find the m where $1 = .004\,641\,6m^{\frac{7}{3}}$, solution is: 10 miles.

His total benefits (revenue) from skiing is then $10(\$1) = \$10$. Total cost to him is average cost at 10 miles multiplied by 10. Average cost is $AC(m,3) = .001\,392\,5m^{\frac{7}{3}}$, so average cost at 10 miles is $.001\,392\,5(10)^{\frac{7}{3}} = 0.30$, and total cost is $10(0.30) = \$3$.

So his net benefits, profits, are $10 - 3 = \$7$.

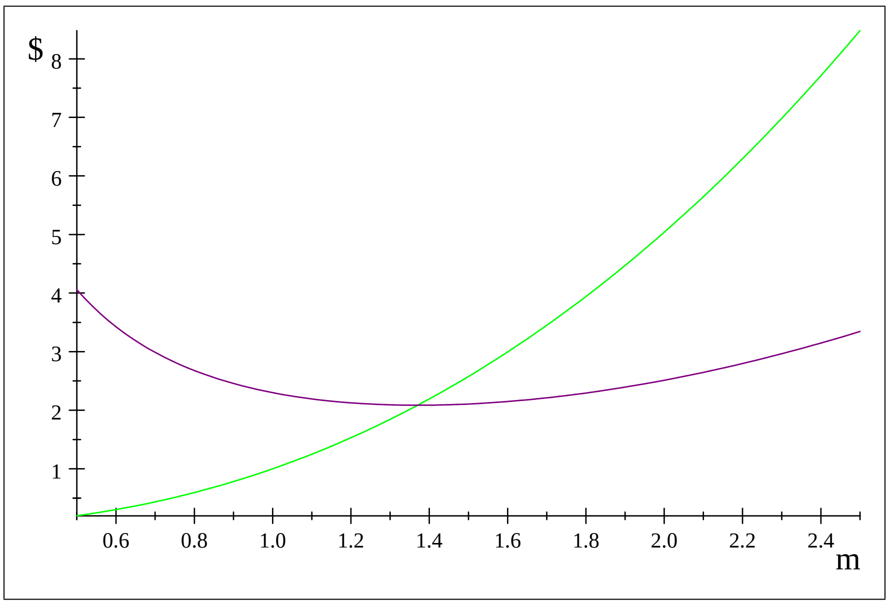
Alternatively, if his parents pay Fred $\$5/mile$, he wants to find the m such that $5 = MC(m)$. he will find the m where $5 = .004\,641\,6m^{\frac{7}{3}}$, Solution is: $19.\,9$ miles.

His total benefits (revenue) from skiing is then $19.9(5) = \$99.\,5$. Total cost to him is average cost at 19.9 miles multiplied by 19.9. Average cost is $AC(m,3) = .001\,392\,5m^{\frac{7}{3}}$, so average cost at 19.9 miles is $.001\,392\,5(19.9)^{\frac{7}{3}} = \$1.\,49$, and total cost is $19.9(1.49) = \$29.\,65$ So his net benefits, profits, are $99.5 - 29.65 = \$69.\,85$

Fred obviously prefers to be paid $\$5/mile$ rather than $\$1/mile$

3.3 In KW, short-run average-cost functions are often presented as U-shaped, not always increasing like Fred's for producing ski miles.

In the book they look like



KW: marginal cost green and average cost prple.

Why the difference between Fred's average cost that is always increasing and average costs being U-shaped in the book?[8]

[8] Note that when marginal cost is below average cost average cost is falling even if marginal cost is rising.

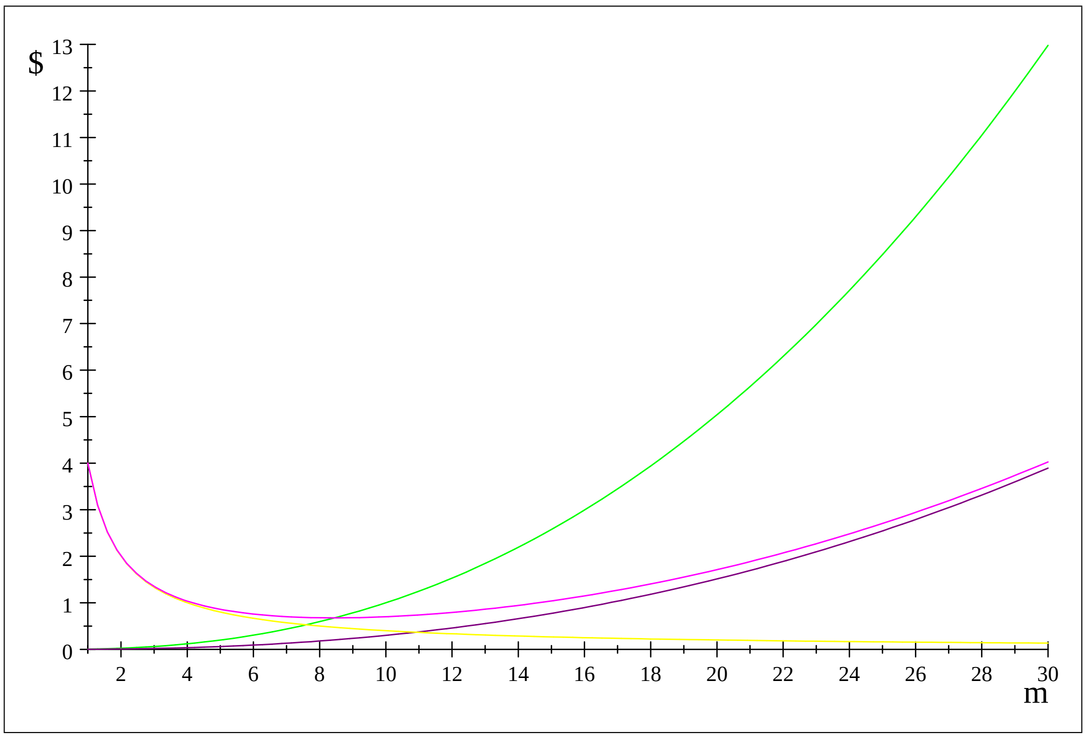
3.3.1 One answer is Fixed Costs?

Fred, up to this point, has no fixed costs. If he skied zero miles there were no short-run costs: his parents paid for his ski equipment.

In KW, the firm incurs a fixed cost even if they produce nothing.

Let's impose a fixed cost on Fred and see how it affects his average cost function for producing ski miles.

Fred's parents decide to make him pay them \$4 every Saturday, no matter whether he skiis or not.[9] Fred then incurs a cost that he must pay no matter how much he does or does not ski. his cost curves then look like:



Fred: grn.=mc, magenta=ac, purple=avc, yell.=ave fixed

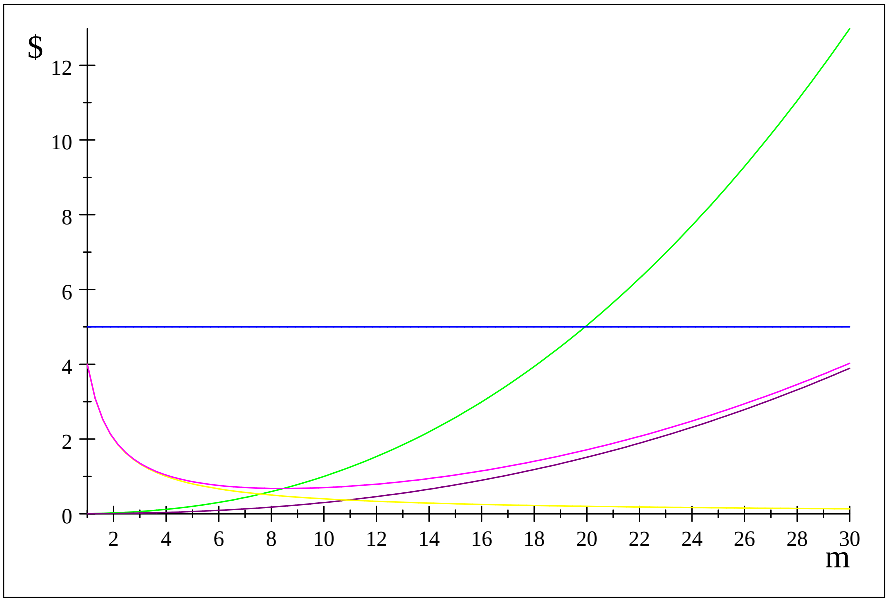
[9] He has to pay his parents back for the new ski boots they bought him at the beginning of the season.

There are two opposing factors affecting Fred's average costs: the more miles Fred skis the smaller the fixed costs per mile (this factor drives average costs down - yellow line declines forever)

And the more miles Fred skis the slower he becomes: average variable costs always increase (this factor puhes average costs up).

Initally, the first effect dominates the second effect causing average cost to fall, but the latter effect eventually dominates the second effect.

Let examine what Fred will do and what he will earn if his parents pay him \$5/mile, as above, but he has to pay the \$4 fixed cost.



Fred: green=mc, magenta=ac, purple=avc, yellow=ave fixed

Does Fred ski more or less than he did when there was no fixed cost? He skis the same amount: 19.9 miles if his parents pay him \$5/*mile*. But his profits decline by \$4. In this case the introduction of a fixed cost did not influence how much he would ski. (Someone should make up an exam question on this point.)

Note that the profit maximizing competitive firm does not minimize average costs; profits are not maximized at that point.

4 Generalize Fred's production to two variable inputs (the quasi-longrun): or what his parents "learned" from Lance Armstrong

why the adjective "quasi"? because there are likely more that two variable inputs in the longrun

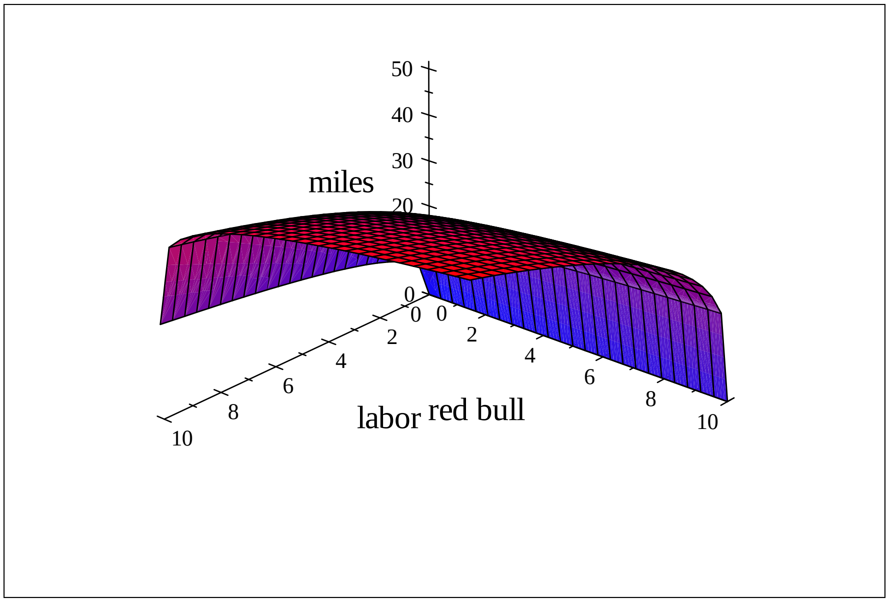
4.1 Fred's three-input production function

Other inputs to consider: cans of Red Bull (caffeine will speed him up) , length and slope of ski run (if it is downhill, Fred will go faster), weather (skis are slower when it is warm or very cold).

To keep things simple, consider three inputs, Fred's labor l, the fixed amount of ski equipment, $\overline{s} = 1$, and cans of Red Bull, r. So, now there are two variable inputs[10], rather than one.

$$m = m(\overline{s}, l, r)$$

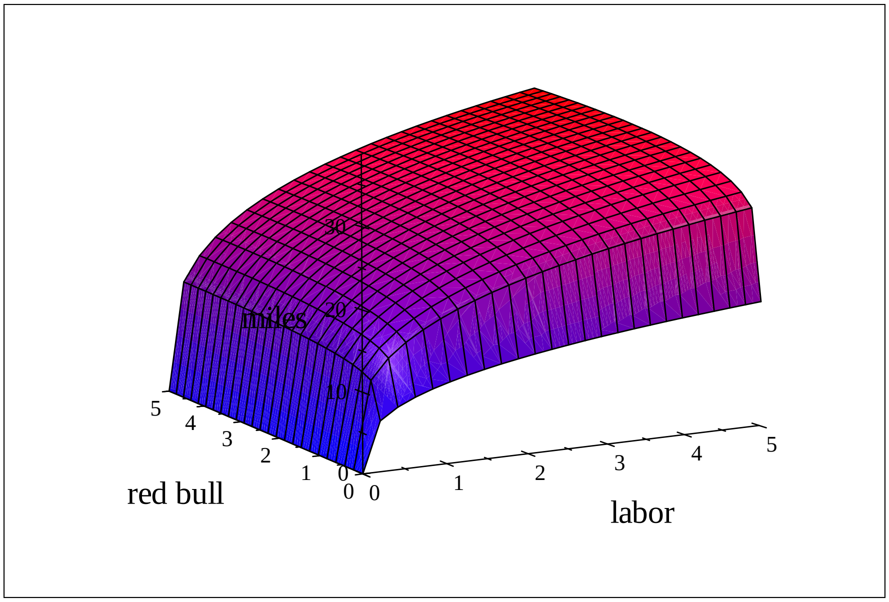
So, it might look like $(m = 10l^{.3}(1 + r^{.2}))$[11] Don't sweat the specific mathematical form, but note that if l or r increases, *miles* increase



Miles skied as a function of time skied and red bull drank.

[10] Variable inputs are inputs that the firm is able to vary in quantity.
[11] Which is our original produciton function if $r = 0$.

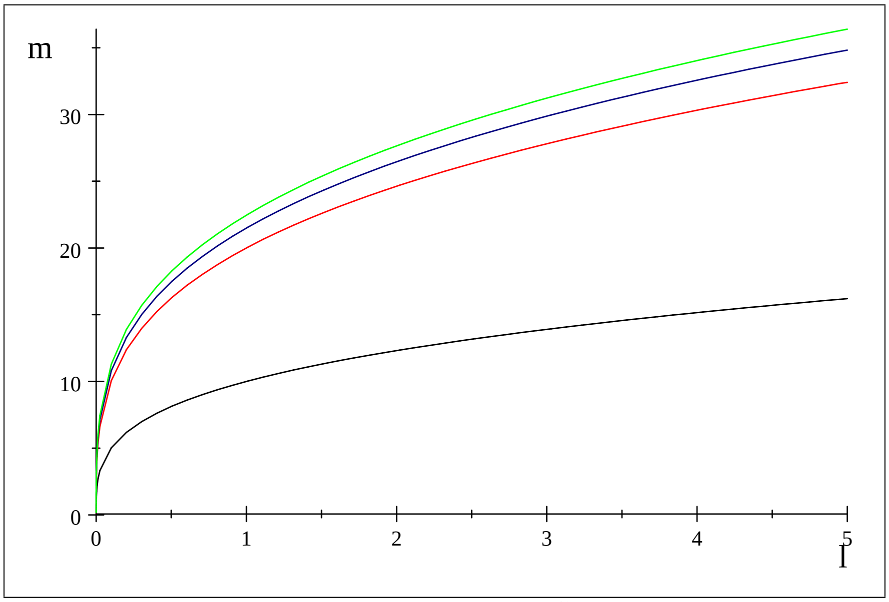
from another angle



Miles skied as a function of time skied and red bull drank.

Note that Red Bull is not an essential input (Fred can ski without it–that is what I initially assumed).

The above graph is a little difficult to figure out, so let me graph the production function for four different amount of Red Bull $r = 0, 1$, 2 and 3 cans. They are as follows:



Blk=0RB, Red=1Blu=2 Grn=3

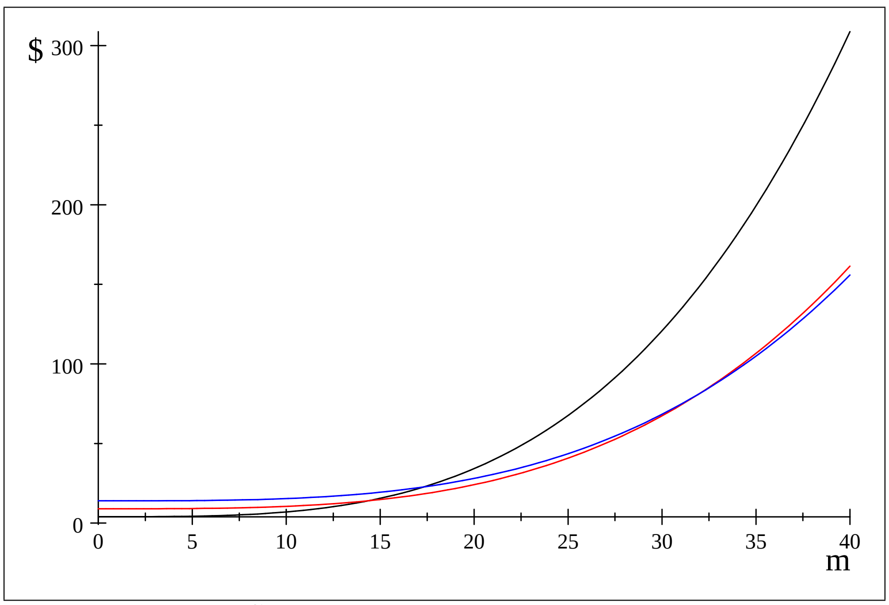
Note how much Red Bull increases Fred's productivity - better than Coke; it increases his marginal product of labor at every level of labor. My initial production function assumed zero cans of Red Bull. (The black line was our orginal production function.)

4.2 Fred's quasi long-run cost function

His parents are cheap so Fred has to pay for his Red Bull, in addition to the $4 fee for being alive on Saturday morning.

So, Fred's total costs of producing ski miles depends on the number of miles he skis, m, the opportunity cost of Fred's time, $w = \$3$ and the per-can cost of Red Bull. Let's assume Red Bull costs $5/can.

$c = c(m, r, 3) = (.001\,392\,5m^{\frac{10}{3}})/(1 + r^{.7}) + 5 * r + 4$ - don't sweat the math; I need it to correctly graph the cost functions.[12]



Cost: black=0 RB, red=1, blue=2

The black cost curve starts at $4; the red at $9 ($4 fixed plus $5 for one Red Bull), and the blue cost curve start at $14

Red Bull shifts down Fred's total cost curve for producing ski miles even though he has to pay for the Red Bull.

[12]The cost function assuming $w = 3$ and $p_r = 5$ as a function of m and r is simply the original cost function divided by how much r bumps m, holding l constant, plus fixed costs of 4 and the cost of r which is r multiplied by 5, by assumption.

4.3 Marginal cost if Fred buys and drinks one Red Bull.

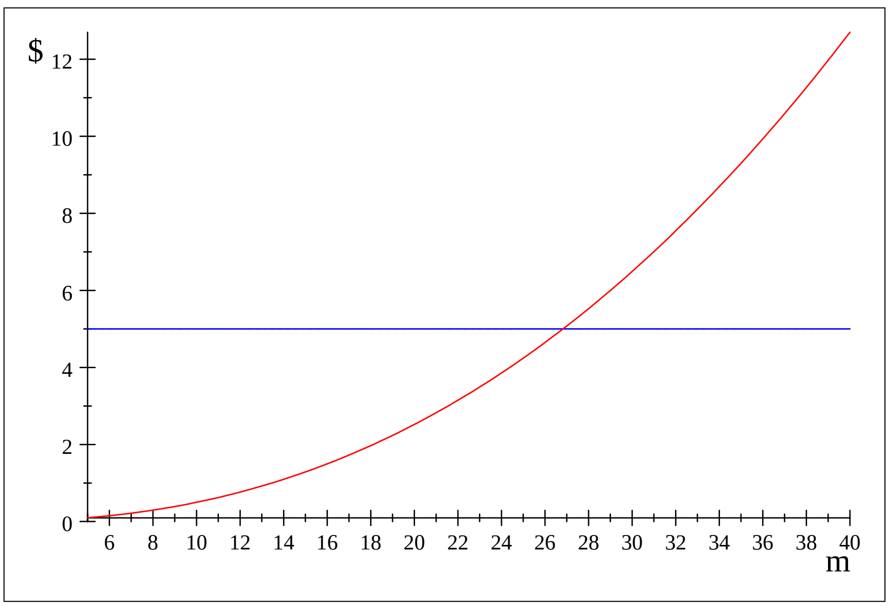
$MC_m(m, r, 3) = (.004.641\,6m^{\frac{7}{3}})/(1 + r^{.2})$ Again, don't sweat the math.[13]

4.4 And his parent pays him \$5/mile

4.5

At the profit maximizing pt. $p_m = 5 = MC_m(m)$; that is profits are maximized when price equals marginal cost.

Assume he drinks one can of Red Bull
Solving $(.004\,641\,6m^{\frac{7}{3}})/(1 + 1^{.2}) = 5$, Solution is: 26.83 miles

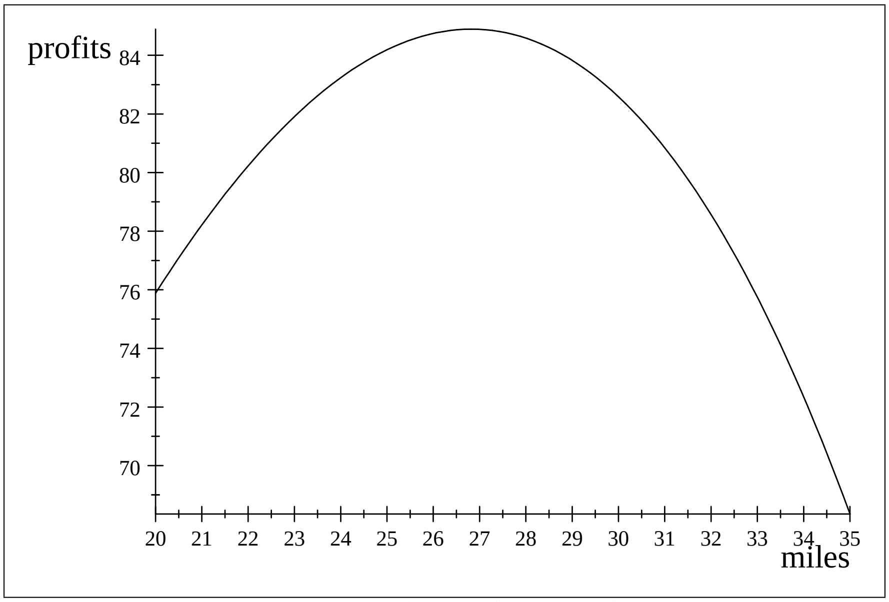


1 Red Bull, marginal cost in red

If he drinks one Red Bull, Fred will ski approximately 27 miles. One can of Red Bull increases his profit-maximizing number of ski miles by approximately 7 miles when $p_m = 5$ Wow – good stuff! Did Lance Armstrong drink Red Bull or did he have something even better.

profits = total revenue - total cost

[13] The marginal cost function is simply the derivative of the cost function wrt m.

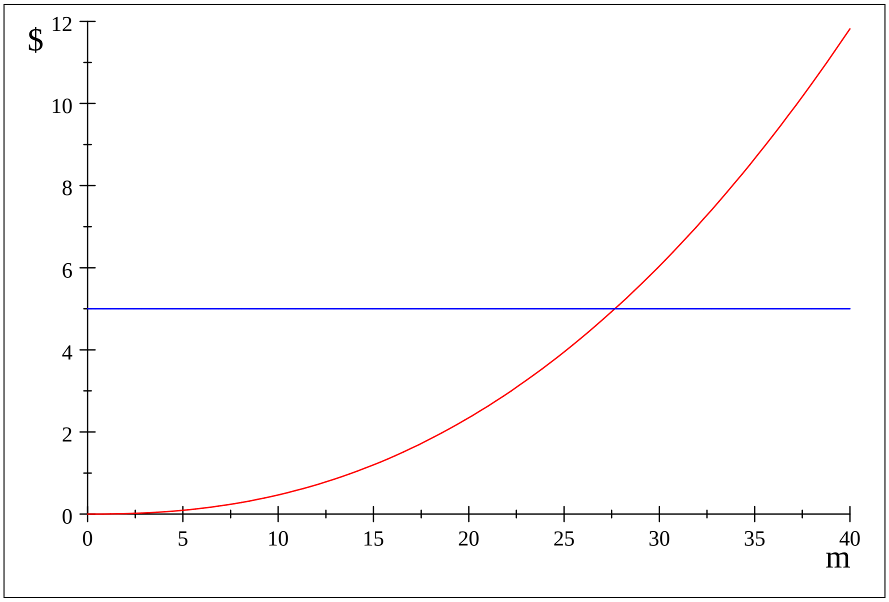


1 Red Bull: Fred's profit as a function of miles skied

Note that Fred's profits were approximately \$70 when he drank no Red Bull, and now they are over \$80 even though the can cost him \$5

4.6 Marginal cost if Fred buys and drinks two Red Bull. And his parents pay him \$5/mile

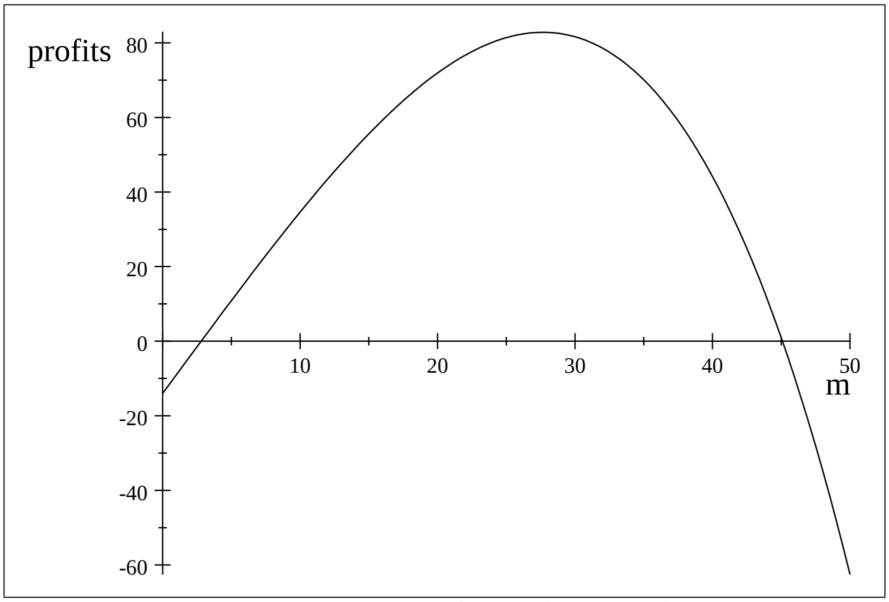
$(.004\,641\,6m^{\frac{7}{3}})/(1+2^{.2}) = 5$, Solution is: 27.664 miles



2 cans Red Bull: marginal cost in red

Fred chooses to ski approximately 28 miles if he drinks two cans of Red Bull, about one mile more than if he only drank 1 can.

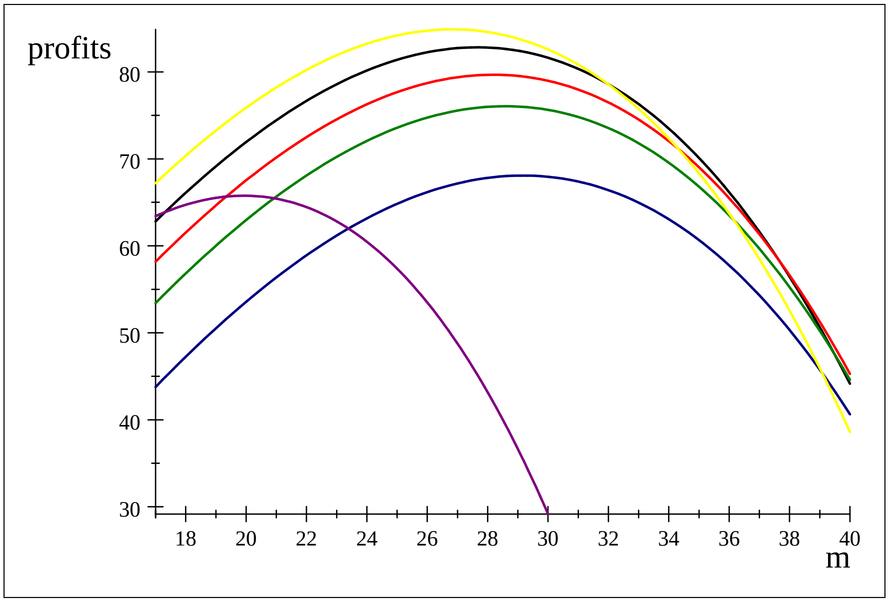
Graphically



2 Red Bull: Fred's profit as a function of m

4.7 How many cans should Fred buy to maximize his profits if he can buy and drink, zero, one, or more cans of Red Bull?

How many cans would he choose to drink?



profit: pl=0RB yl=1 blk 2, rd 3, grn 4, blu 6

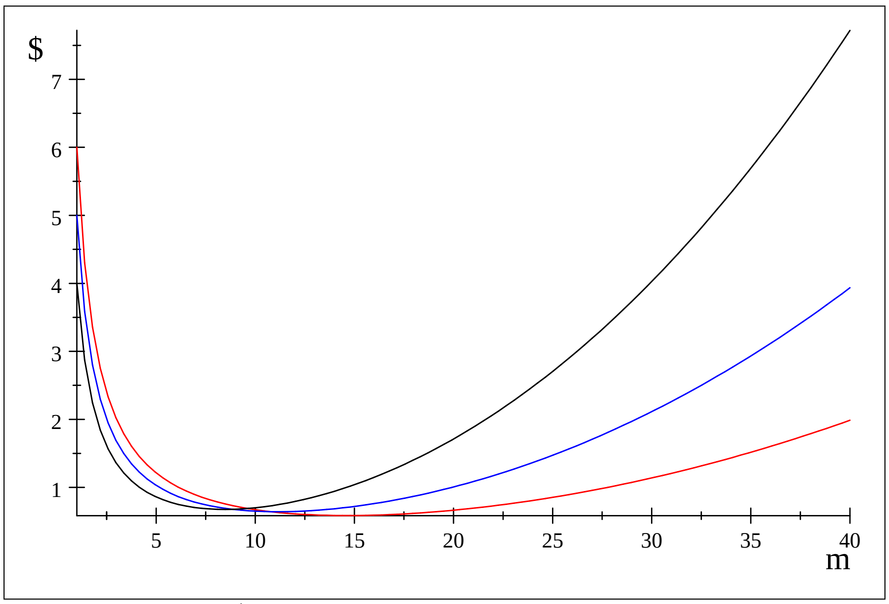
Fred wants to buy and drink one can of RB.

4.8 Distinguishing between the shorter and longer run

In the longer run, Fred gets to choose both the amount of time he will ski and the number of Red Bulls he will drink.

In the shorter run, Fred gets to choose the amount of time he will ski and is constrained to drink a certain number of Red Bull. (in my initial example it was zero cans)

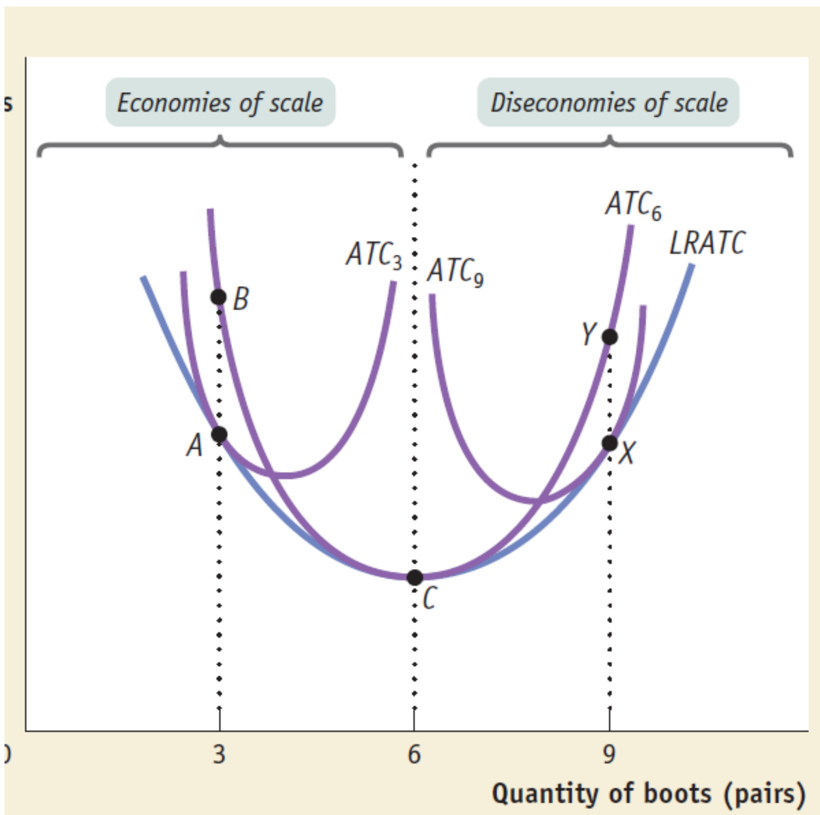
Let's look at Fred's average cost curves for zero, one and two cans of Red Bull



Av. cost red=3 RB, blue=2,black=1

The longrun average cost curve (not shown) is traced out by the bottom of the three curves.

The following graph from KW shows, more clearly, the envelope nature of the long-run average cost curve.


s
Economies of scale
Diseconomies of scale
ATC_3
ATC_9
ATC_6
$LRATC$
B
A
C
Y
X
0
3
6
9
Quantity of boots (pairs)

4.9 Fred's production function AND his ISOQUANTS

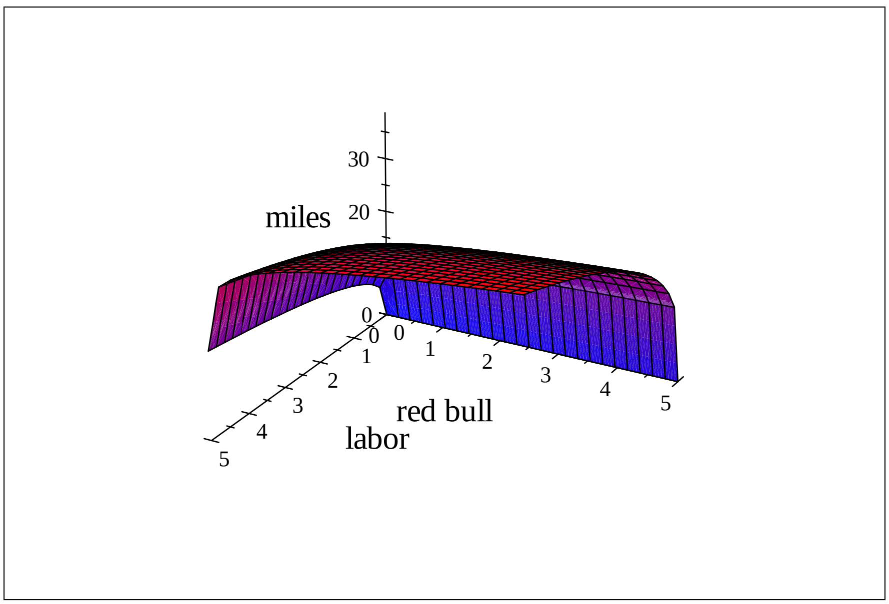
Above we looked at Fred's production function:

Let's look at it again.

Consider three inputs, Fred's labor l, the fixed amount of ski equipment, $\overline{s} = 1$, and cans of Red Bull, r. Assume two variable inputs.

$$m = m(\overline{s}, l, r)$$

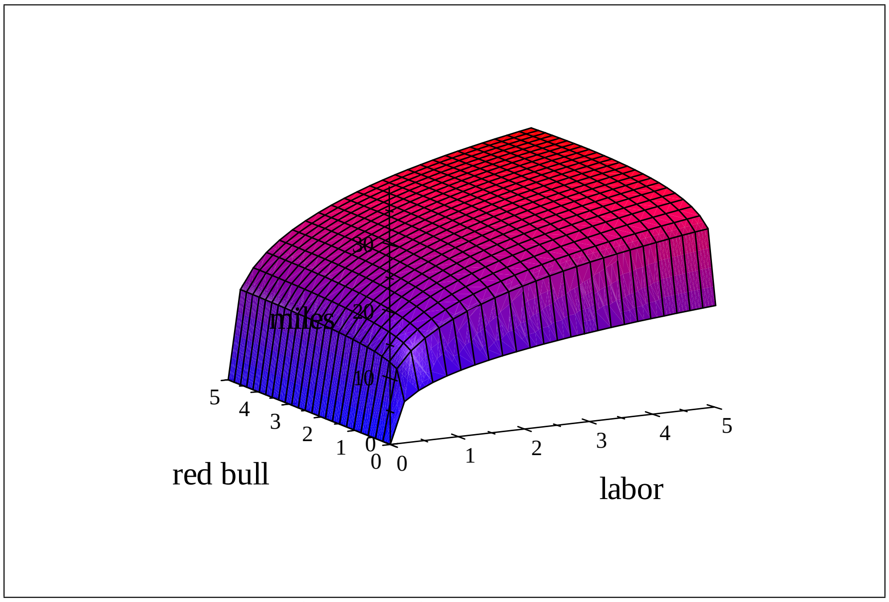
So, as above, it might look like $(m = 10l^{.3}(1 + r^{.2}))^{14}$



Miles skied as a function of time skied and red bull drank.

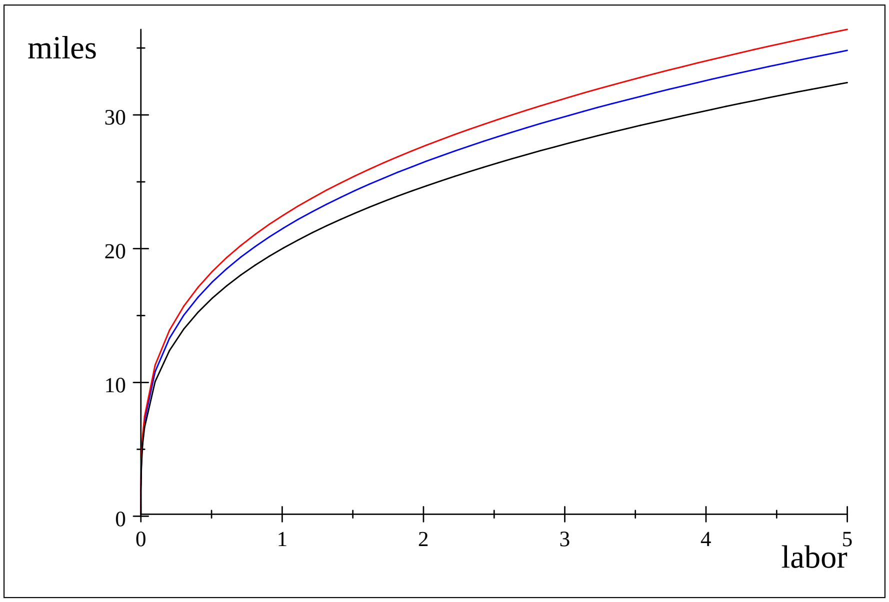
from another angle

[14] Which is our original produciton function if $r = 0$.



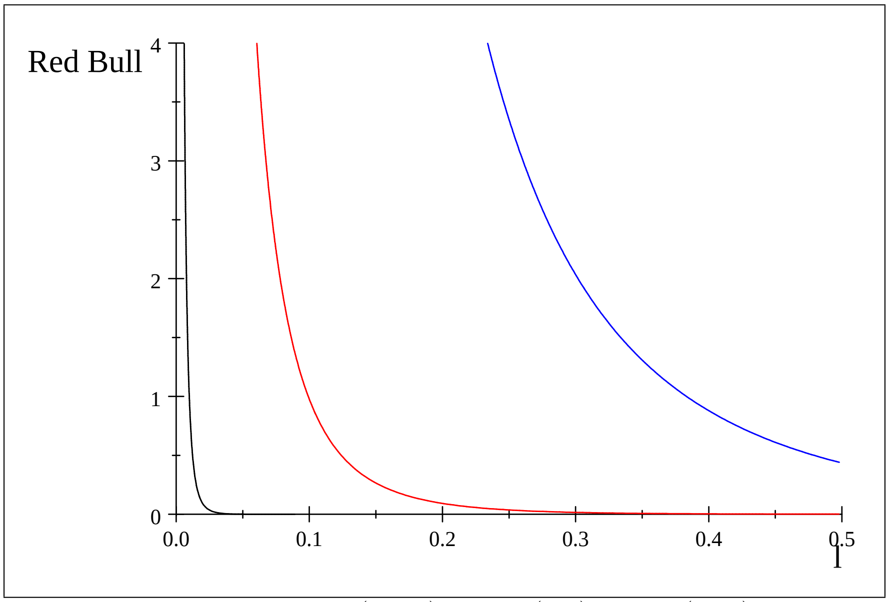
Miles skied as a function of time skied and red bull drank.

Above, we graphed it for Red Bull $r = 1$, 2 and 3 cans. They were as follows:



Production of miles, black=1 Red Bull, blue=2, and red=3 cans

Now his parents want to slice the production function another way- horizontal slices rather than vertical slices. Let's look at all those combinations of l and r that produce the same maximum number of ski miles



Isoquants: 5 miles (black), 10 m. (red), 15 m. (blue)

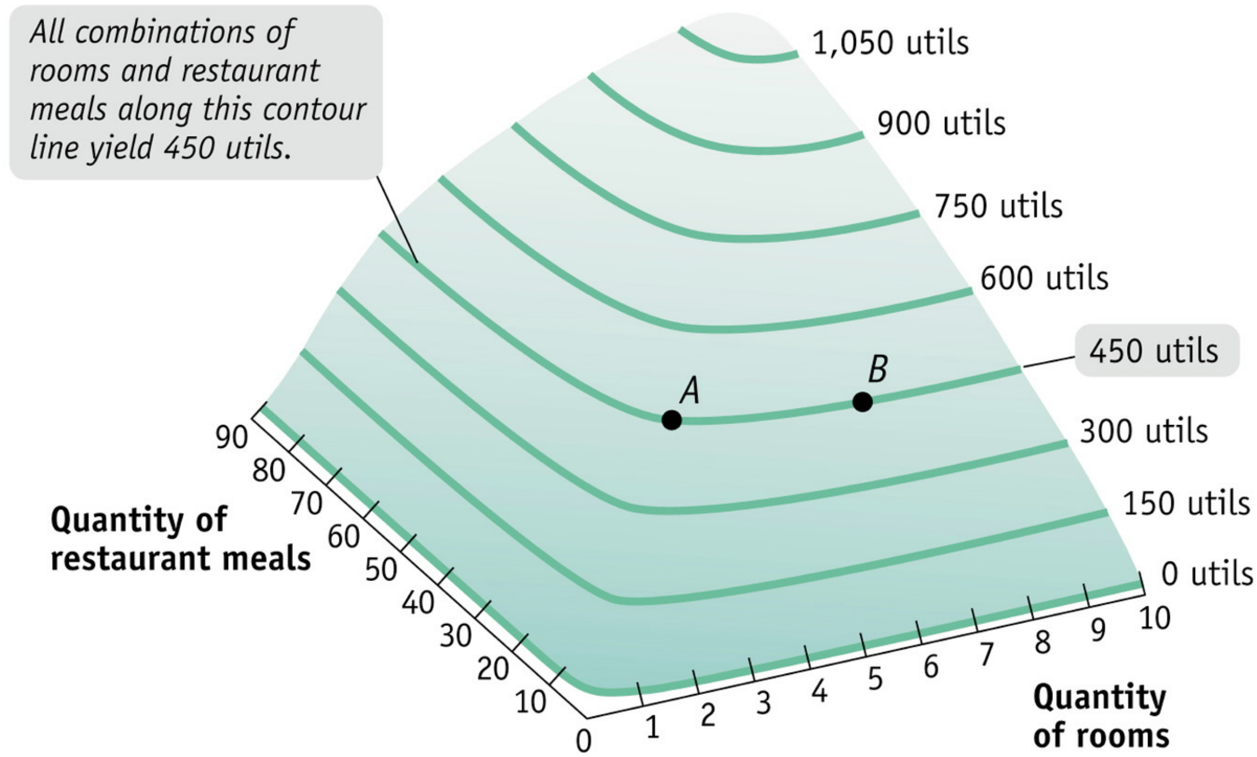
These are called isoquants: "quant" as in "quantity", and "iso" as in "one": every point on an isoquant corresponds to the same maximum quantity.

Note that one can produce a specific quantity without producing it using an input combination that is on the isoquant for that quanity. That is, one can be wasteful and use more inputs than needed.

The isoquants show how Fred can substitute labor for Red Bull in the production of ski miles.

The slope of these isoquant curves identify the rate at which one **input** can be substituted for the other in the production of ski miles

The negative of the slope of the isoquant (r on the vertical axis, l on the horizontal axis) is called the *marginal rate of technical substitution of labor for Red Bull*, $MRTS_{lr} = -\frac{\triangle r}{\triangle l}|_{\triangle m=0}$. The $MRTS$ is to the production function as the MRS is to the indifference curve.



4.10 The utility function is a type of production function

Now his parents want you to rethink the utility function. Think of a utility function as a production function: one consumes goods to produce utility, just like the firm uses inputs to produce output.

An isoquant identifies all combinations of inputs that produce the same output level

An indifference curve identifies all combinations of goods that produce the same utility level (same output of utility).

The slope of an isoquant identifies the rate at which one input can be substituted for another holding output constant. It depends completely on the state of technical knowledge for producing the good.

The slope of an indifference curve identifies the rate at which the individual will substitute one input for another holding utility constant. It depends completely on preferences. Preferences are what determines how an individual produces utility.

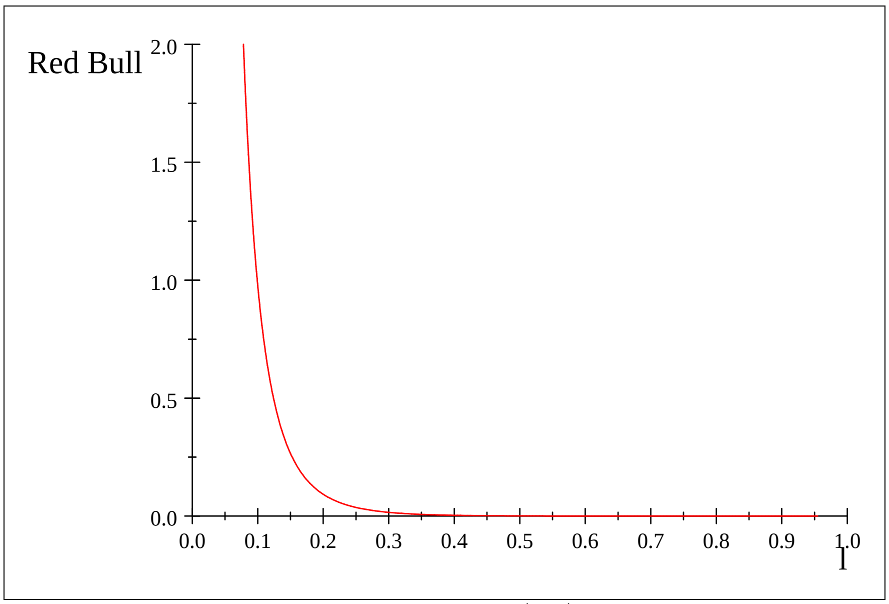
4.10.1 Profit maximization implies that whatever output level is produced, it is produced at minimum cost[15]

Consider Fred in the longrun: he can vary both the amount of time he can ski, l, and the how much Red Bull he drinks, r.

Imagine he chooses to ski 10 miles.

What combination of l and r will he choose to produce those 10 miles of skiing?

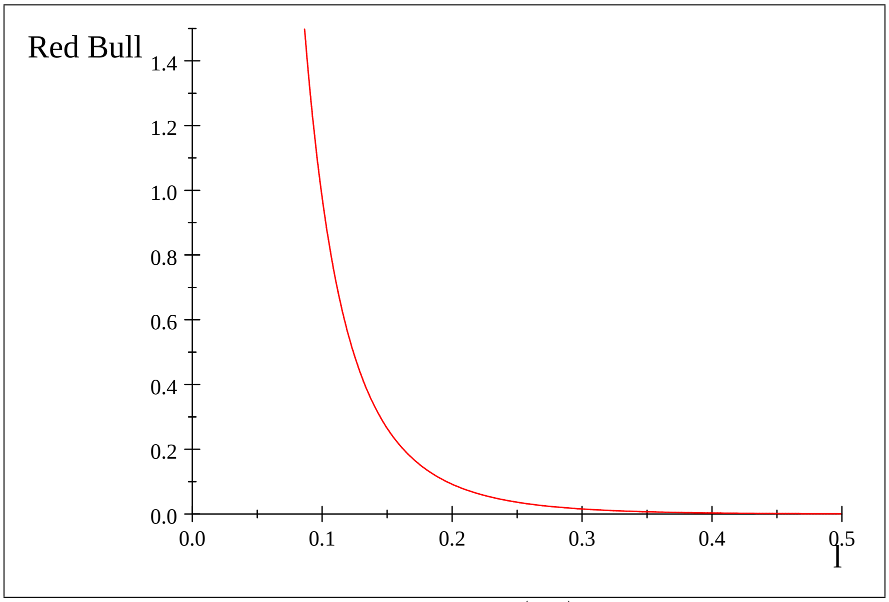
Recollect there his isoquant for producing 10 miles of skiing is



Isoquants: 10 m. (red)

Or drawing it to a different scale.

[15]But minimizing the cost of producing whatever amount is produced does not imply that profits are maximized – the firm might not be producing the profit-maximizing level of output.



Isoquants: 10 m. (red)

Also recollect that Red Bull costs a \$5 can and he values an hour at \$3

Given these input prices, what are the different combinations of time and Red Bull that he can buy for \$$v$?

$$v = 3l + 5r$$

identifies all those combination of l and r that cost v dollars.

Solving for r, $r = \frac{v}{5} - \frac{3}{5}l$

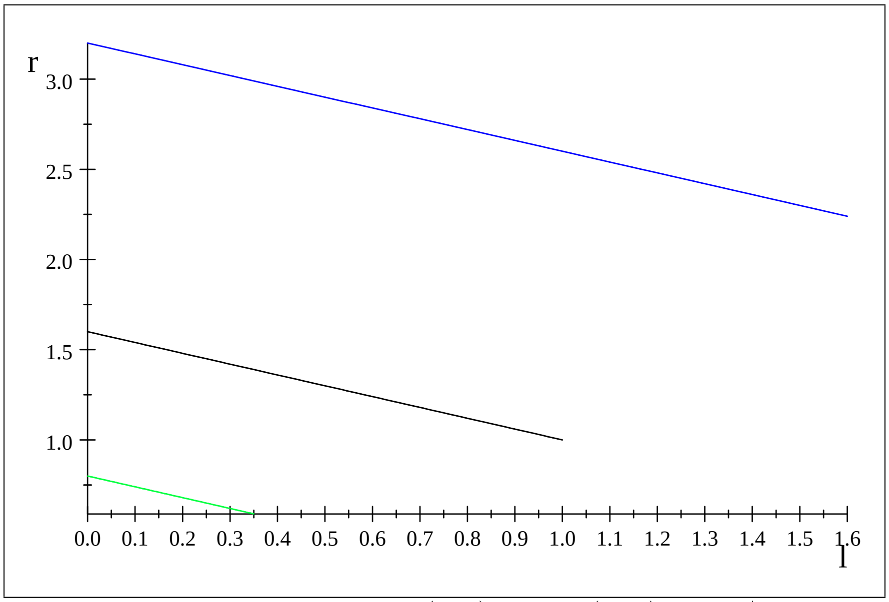
In explanation if the firm, Fred, spends all of it his money v, on Red Bull, he can buy $v/5$ units, but, for every hour he works, Red Bull intake must decline by $3/5$ units.[16]

This line, $r = \frac{v}{5} - \frac{3}{5}l$, is called an isocost line ("iso" as in "equal"). **It is just like the consumer's budget line: indentifying the different combinations of stuff that can be purchased for a same (equal) amount of money.**

The slope of the isocost line is the rate at which the **market allows** the **firm** to substitute one **input** for another

For example, if $v = 3$, $r = 3 - 3l$ and if $v = 5$, $r = 5 - 3l$. Graphing these

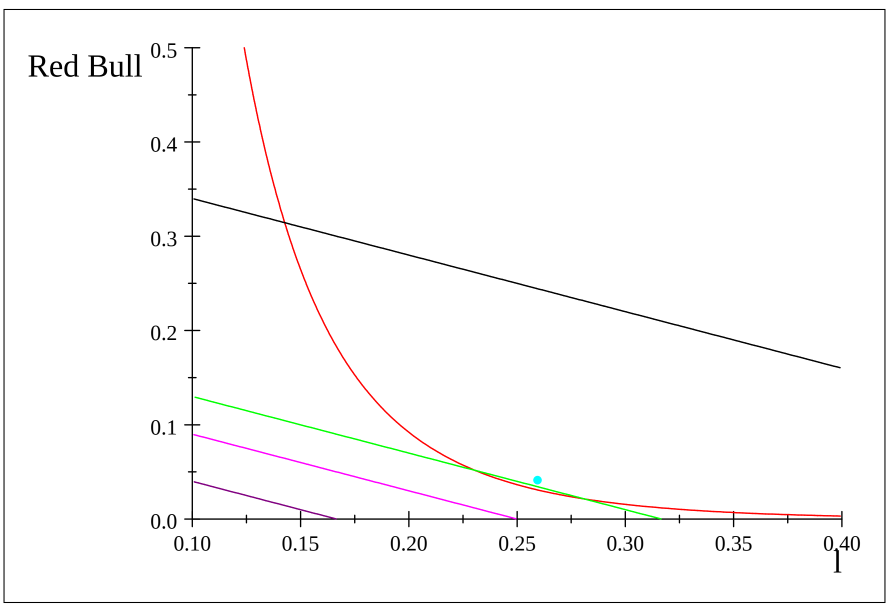
[16] Think of Fred as working for herself, and paying herself \$3/hour.



Fred's isocost lines: $v = 8$ (blk.), $v = 16$ (blue), green \$4

The slope of each of these lines is $-\frac{3}{5}$. If he skis one more hour he will have to pay herself \$3, so have \$3 less to buy Red Bull, so have to reduce his consumption or Red Bull by 3/5 of a can.

If Fred want to ski ten miles, and wants to minimize the cost of doing it, what combination of time and Red Bull should he use to produce the 10 miles?



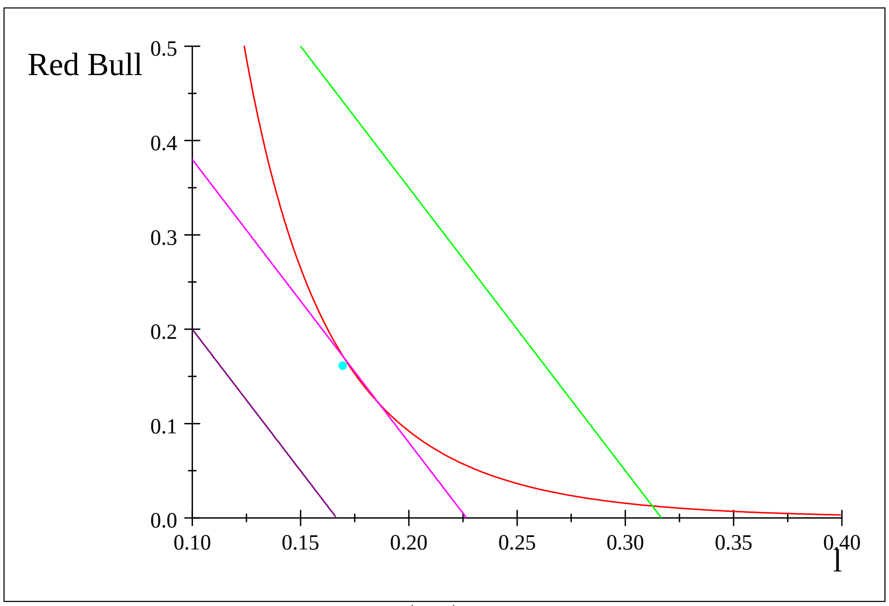
Isoquants: 10 m. (red) with 4 isocost lines

Black is Fred spending \$2, green \$1, magenta \$0.75 and purple \$0.50.

It looks like, given the input prices, the cheapest way for him to crank out 10 miles is to buy and drink only a sip of Red Bull and then ski for around 16 minutes (.26 hours). The cheapest way cost approximately \$0.95.

Note that at the cost-minimizing input combination, the price ratio of the two inputs (the minus of the slope of the isocost line) equals the $MTRS_{lr}$ in the production of ski miles.

Look what happens if the price of Red Bull is \$1 rather than \$5

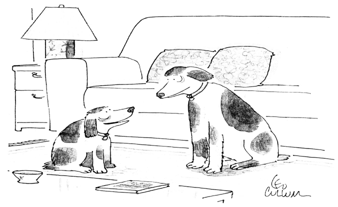


Isoquants: 10 m. (red) with 4 isocost lines

The minimum cost of producing the 10 miles decreases from approximately 95 cents to 68 cents because Fred now chooses to buy more Red Bull so he can spend less time skiing the 10 miles. His cost-minimizing input mix changed because the price of RB declined.

The take home message here is there are lots of ways to skin a cat, and, if one want to make a profit skinning cats, for the chosen production of cats without skin, one wants to choose the cost-minimizing input combination.

The caption is "Just how many ways are there to skin a cat?"



"Just how many ways are there to skin a cat?"

Since there are often many ways to produce a given amount of something, the way it will be produced is strongly influenced by the input prices.

So, if input prices vary across time or space, the same output will not always be produced in the same way: the cost-minimizing way to produce x units of output will vary with input prices.

For example, consider two inputs labor, l, and capital, k in the production of food.

Draw an isoquant for a given amount of food production, labor on the horizontal axis.

The minimum cost way of producing that amount of food with depend on the relative price of labor and capital $\frac{w}{p_k}$ where w is the price of labor, the wage rate. $\frac{w}{p_k}$ is the slope of the isocost line.

If $\frac{w}{p_k}$ is low (labor is cheap relative to capital) food will produced using a lot of labor relative to capital (labor intensively). This is what you see in poor developing countries with lots of labor (people) but relatively little capital.

Draw an isocost line $m = wl + p_k k$, implying $k = \frac{m}{p_k} - \left(\frac{w}{p_k}\right) l$

If $\frac{w}{p_k}$ is high (labor is expensive relative to capital) food will produced using a lot of capital relative to labor (capital intensively). This is what you see in countries like the U.S.

Draw with graphs with both the isoquant line and some isocost lines.

Think about a traditional Chinese meal vs. a traditional American meal. Joe America throws a big hunk of meat on his big-ass gas grill, and has his capital and meat-intensive dinner ready in 15 minutes. Joe China takes a bunch of time to chop a small amount of meat and vegtables into a bunch of small pieces.

Consider another example: two inputs, oil and other inputs (insulation, weather stripping, etc.) in the production of heating your house. A T.A. should make up a question about this.