1 A map of indifference curves are a visual way of representing preferences

draft Oct 30, 2017

Remember that consumer theory assumes you have a ranking of bundles (*your* preferences) such that

$\forall$ bundles j and k either

- $\mathbf{x}^j \succ \mathbf{x}^k$ read this, bundle j is ranked higher than bundle k
- $\mathbf{x}^j \prec \mathbf{x}^k$

or

- $\mathbf{x}^j \sim \mathbf{x}^k$

Consider any bundle, e.g. $\mathbf{x}^m$. Picture it as a particular point in n-dimensional space, where n is the number of commodities in our bundle.

Then identify all those bundles such that you are indifferent between each of the those bundles and $\mathbf{x}^m$

(Note that if $\mathbf{x}^j \sim \mathbf{x}^m$ and $\mathbf{x}^k \sim \mathbf{x}^m$ then $\mathbf{x}^j \sim \mathbf{x}^k$: that is you are indifferent between all the bundles in this set).

That is, we can group bundles into different sets (*indifference sets*) such that you are indifferent between all of the bundles in each of the indifference sets, but never indifferent between two bundles in different indifference sets.

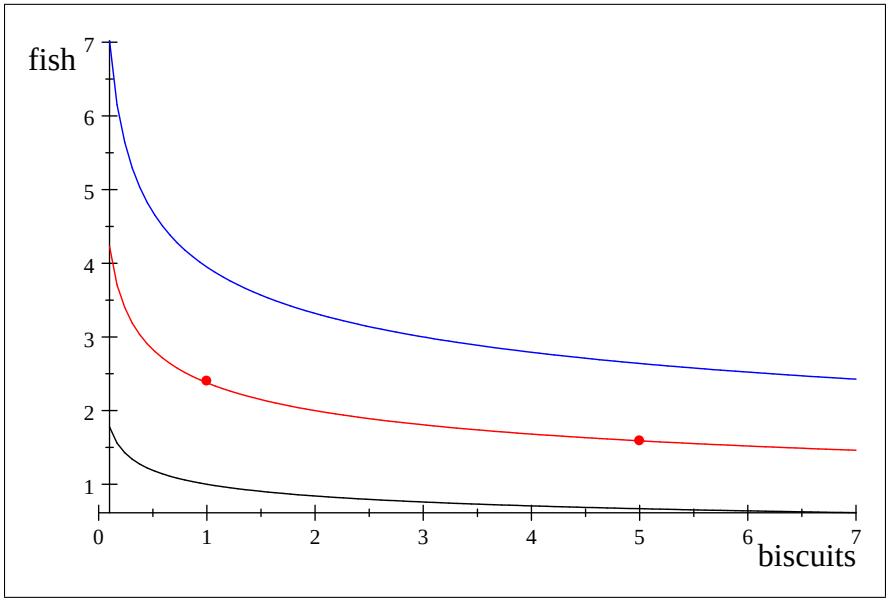
(Note that if $\mathbf{x}^j \succ \mathbf{x}^k$ or $\mathbf{x}^j \prec \mathbf{x}^k$, they cannot, by definition, be in the same indifference set.)

Indifference sets are easy to visualize is we assume there are only two commmodities in the world, x_1 and x_2 because in this case a bundle is defined in terms of its levels of just these two commodities.

Using an example I will use later as well, I am going to assume two commodities: fish and biscuits, and that, for me, both are goods (for me, more of each is always preferred).

Assume the following represents my preference for acquarium fish (in my aquarium) and biscuits for me and my dog Sofie.[1]

[1] I have a small trust fund that I use to buy fish and biscuits. Sofie and I have no other needs.



Three indifference curves

I am indifferent between every bundle on the black line; am indifferent between every bundle on the red line, and am indifferent between every bundle on the blue line.

For example I am indifferent between 1 biscuit with 2.4 fish, and 5 biscuits with 1.6 fish: each of these bundles is on my red line.

Notice how my indifference curves become flatter when there are relatively more biscuits than fish in the bundle (steeper as there are relatively more fish in the bundle). It does not have to be this way, but often it is this way.

If a bundle is not on my blue line, I rank it ranked either higher or lower than every bundle on the blue line. The same holds for my black line, my red line, and every other indifference line.

Each of these lines is called an *indifference curve*. An indifference curve, for me, identifies all of the bundles between which I are indifferent.

In the above example (since fish and biscuits are assumed to be goods), I strictly prefer bundles to the right of (above) an indifference curve to all the bundles on that indifference curve, and bundles to the left of (below) an indifference curve are ranked strictly lower than all of the bundles on that indifference curve.

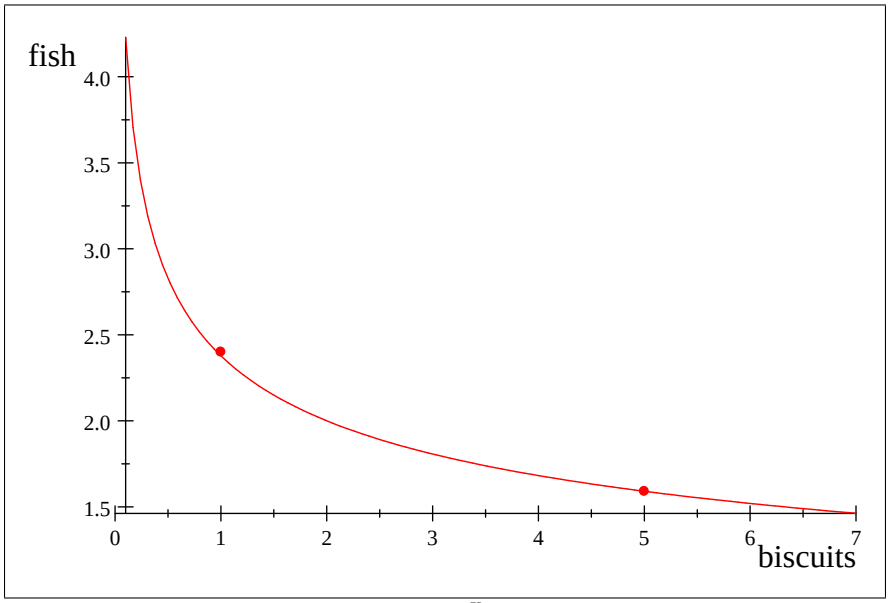
Note that every bundle is on some indifference curve.[2]

Your indifference curves contain a lot of valuable information about your preference.

And, they give us a visual way of visualizing you highest-ranked affordable bundle.

[2]Note that an indifference set could, in theory, of only one bundle. That is it is a "dot" not a curve. If this was thas the case every other bundle would be ranked strictly higher or strictly lower than this bundle. While this is possible, it is highly unlikely.

Now let's look at my red indifference curve in more detail

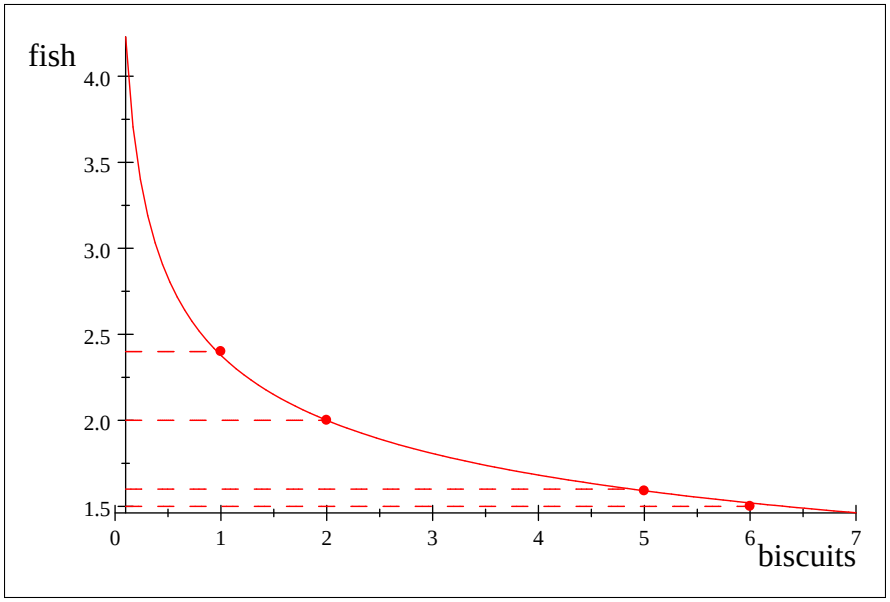


Edward's red indfference curve

Which indicates, as noted above, that I am indifferent between 1 biscuits with 2.4 fish, and 5 biscuits with 1.6 fish.

Said another way, to increase our biscuit consumption from 1 to 5 (an increase of 4 biscuits) I would be willing to give up .8 fish $(2.4 - 1.6)$

1.0.1 Let's do it again but for two smaller changes in the number of biscuits



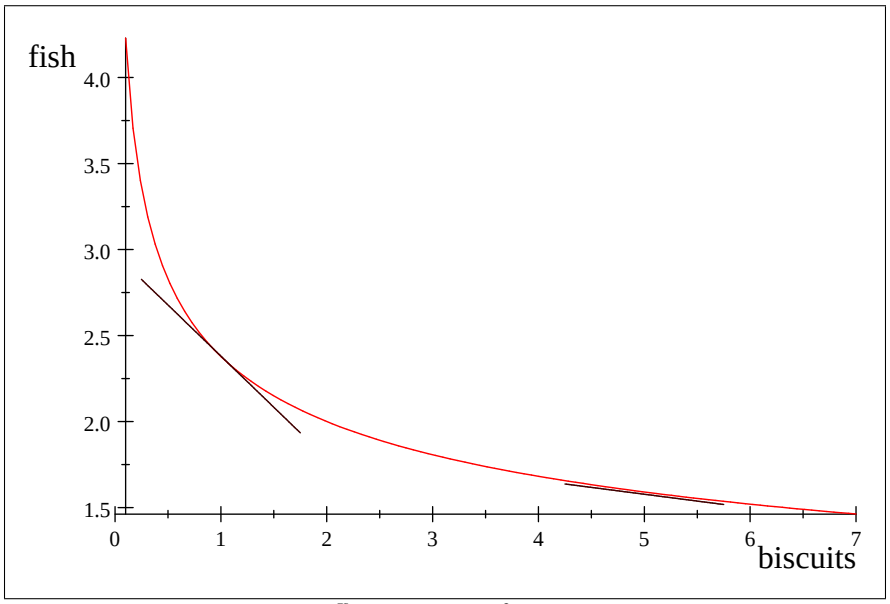
Indifference curve for $u = 2$

If we currently have 1 biscuit and 2.4 fish, my wtp to get another biscuit is .4 fish. If currently 5 biscuits and 1.6 fish, my wtp get another biscuit for Sofie is only .1 fish.

So, for me, the more biscuit-intensive my bundle, relative to fish, the fewer fish I'm will to go without to get another biscuit.

1.1 Consider the slope of my red indifference curve.

Now I redraw the same red indifference curve adding two tangent lines: one at $b = 1$ and one at $b = 5$. These tangent lines represent the slope of the indifference curve at $b = 1$ and at $b = 5$. Notice that the tangent is steeper (has a more negative slope) at $b = 1$. That is, for this indifference curve, the slope becomes less negative as b increases



Indifference curve for $u = 2$

At dog biscuits $= 1$ the slope is $-.6$, and at dog biscuits equal $= 5$ the slope is $-.08$. Put loosely, when Sofie and I are consuming 1 biscuit, I would give up .6 fish to get another biscuit, but when we are consuming 5 biscuits I would only give up .08 fish to get another biscuit.[3]

[3] The answer are slightly different from above because, the slope is declines as one increases biscuits from 1 to 2, and from 5 to 6.

1.2 wtp (willingness to pay) and MRS (marginal rate of substitution)

The slope of my indifference curve, at a point, is

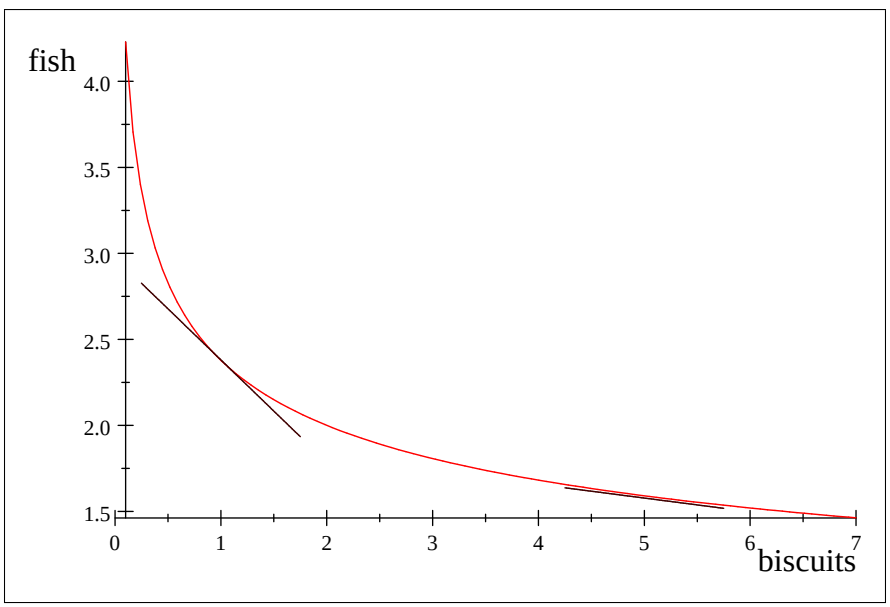
$$\left.\frac{\triangle f}{\triangle b}\right|_{maintaining\ indifference}$$

The negative of the slope of the indifference curve[4], at a specific amount of dog biscuits, is, approximately, how many fish I am willing to give up to get one more dog biscuit.

That is, the slope, in absolute terms, is my willing-to-pay, wtp, for an additional biscuit in terms of forgone fish.

$$-\left.\frac{\triangle f}{\triangle b}\right|_{maintaining\ indifference} = wtp_b$$

Looking back at my red indifference curve , if I am currently consuming 1 dog biscuit I am willing to pay approximately .4 fish (change my fish consumption by $-.4$) to get one more biscuit. Alternatively, if I am currently consuming 5 biscuits I am only wtp .1 fish to get on more biscuit.



Indifference curve for $u = 2$

[4] remember that if both commodities are goods the indifference curve will have a negative slope, and the negative of a negative is a positive.

As the relative amount of biscuits in my bundle increases, my *wtp* to get more biscuits, in terms of forgone fish, declines.

If the indifference curve is a straight line, along that line (of indifference) *wtp* to get more biscuits in terms of forgone fish is a constant.

1.2.1 What is my wtp_f , my wtp for additional fish in terms of forgone biscuits?

If my wtp for an additional biscuit is .5 fish, then my wtp for an additional fish is $\frac{1}{.5} = 2$ biscuits, as it must. (one is the inverse of the other.)

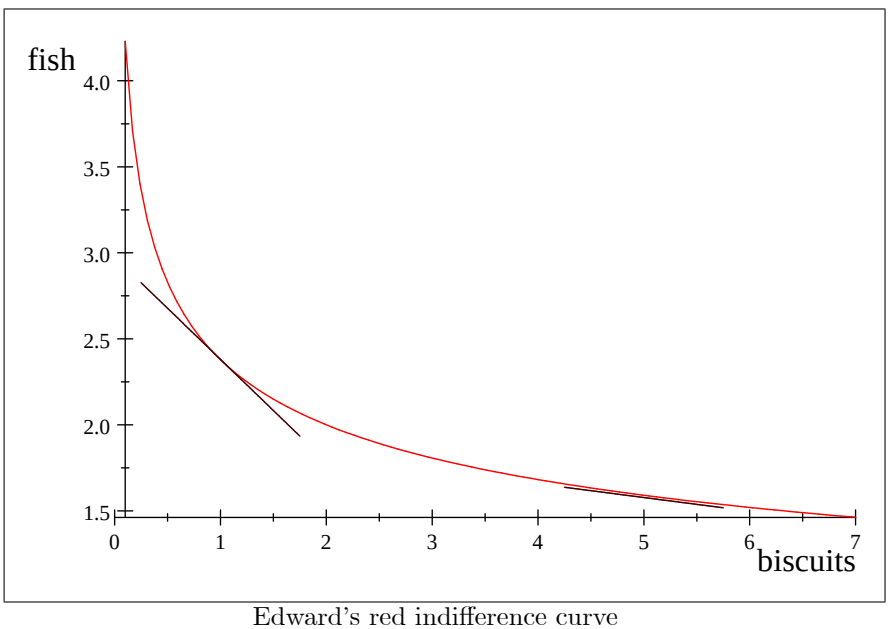
Another name for *wtp* for an additional biscuits is the *marginal rate of substitution of biscuits* ***in place of (for)*** *fish*, $MRS_{bf} = -\frac{\triangle\ fish}{\triangle\ biscuits}\mid_{\ maintaining\ indifference} \equiv wtp_b$

what you will pay to get another biscuit, in terms of fish.

Another name for *wtp* for an additional fish is the *marginal rate of substitution of fish* ***in place of (for)*** *biscuits*, $MRS_{fb} = -\frac{\triangle\ biscuits}{\triangle\ fish}\mid_{maintaining\ indifference} \equiv wtp_f$

what you will pay to get another fish, in terms of biscuits

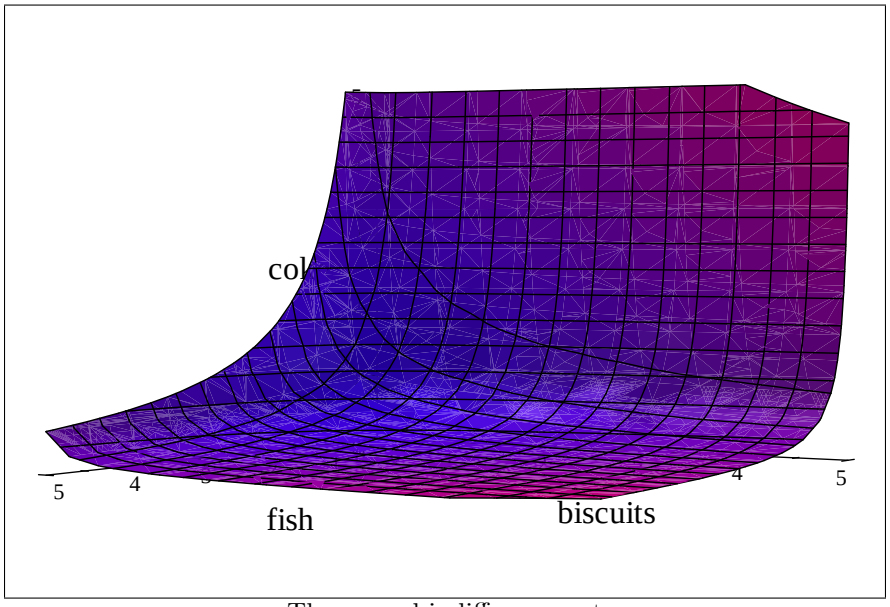
$MRS_{bf} = -\frac{\triangle\ fish}{\triangle\ biscuits}\mid_{maintaining\ indifference}$ is the negative of the slope of the indifference curves, so a positive number if the indifference curve in negatively sloped. (Note that indifference curves for "goods" are negatively sloped.)



Edward's red indifference curve

1.3 What would an indifference curve (set) look like if there were three rather than two goods: (fish, biscuits, and Diet Coke)

It might look something like this (sort of like a satelite dish)



Three good indifference set

1.4 An important aside: consider a world of two commodities: *goods* and *pollution*, where goods are, by definition, good and pollution is bad.

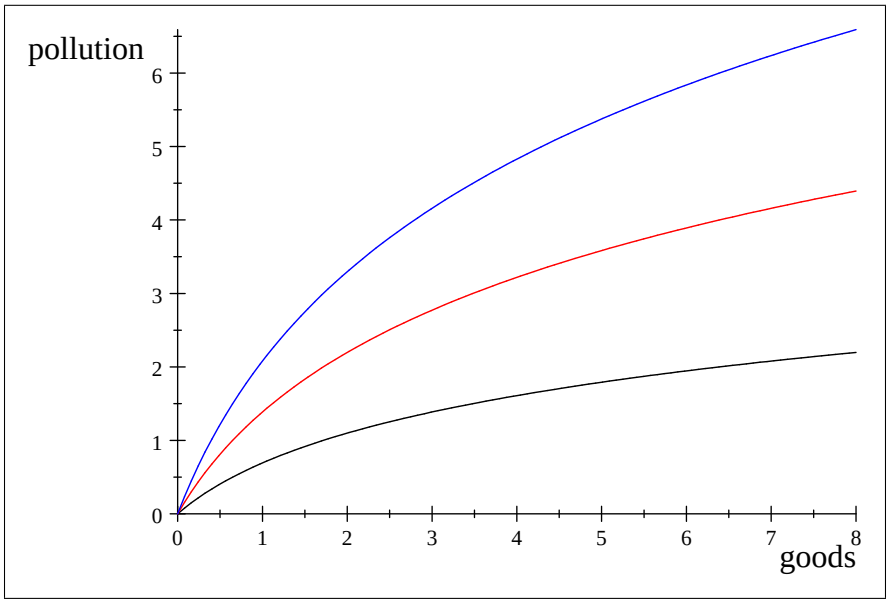
Remember that I am an environmental economist, so like environmental examples.

Draw a representative individiual's indifference curves with goods on the horizontal axis and pollution on the vertical axis.

Assume everyone has the same preferences.

T.A.s are making up problems where one commoditiy is a good and one is a bad.

The following are three **indifference** curves, each has a positive slope (these are **not** three different utility functions).



Three indifference curves

In what direction are the bundles higher ranked?

If one goes straight up (increasing pollution, holding goods constant), one experiences lower ranked bundles

If one moves straight down (decreasing pollution, holding goods constant), one experiences higher-ranked bundles

If one moves straight right (increasing goods, holding pollution constant) one experiences higher-ranked bundles.

If one moves straight left (decreasing goods, holding pollution constant) one experiences lower-ranked bundles.

If one moves to the southeast (more goods, less pollution), one experiences higher-ranked bundles.

If one moves to the northwest (fewer goods, more pollution), one experiences lower-ranked bundles

Remember that there is indifference between all the combinations of pollution and goods on an indifference curve. Bundles on the black line are ranked

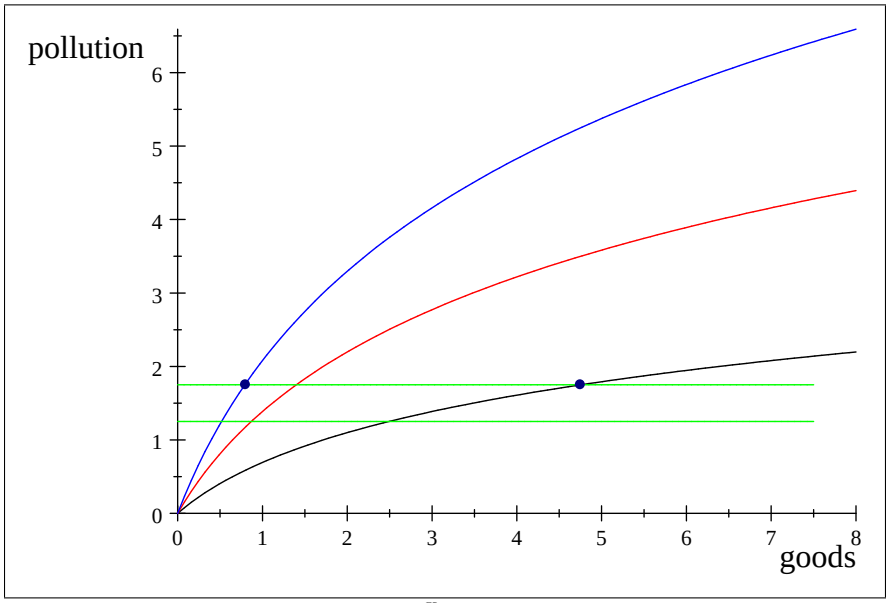
higher than bundles on the red line, and bundles on the red line are ranked higher than bundles on the blue line.

Think about wtp and MRS when one of the commodities is a good and one is a bad. E.g. what is your wtp for another unit of pollution? and what is your wtp to get rid of a unit of pollution? Are these amount positive or negative, and what determines their magnitude?

Your wtp to get more of the good is the additional amount of pollution you would accept.

Your wtp to get another unit of pollution is how much more goods you would have to be given to accept that additional unit of pollution, so in it is actualy your wta (willingness-to-accept) additional pollution, your wtp is negative, so a wta.

Imagine a poor individual, for example one in China. on the blue indifference curve, in contrast to a rich Boulderite on the black one. I chose China because they, like the U.S. are a major polluter, and average incomes are much lower in China than in Boulder.



Three indifference curves

I am assuming both individuals have the same preferences (same indifference map)

I choose the inital bundles so that both experience the same amount of pollution but the guy in Boulder is consuming a lot more goods

How much would the Chinese guy pay, in terms of fewer goods to decrease pollution from 1.75 to 1.25 units (a decrease of .5 units of pollution)? (Remember he is on the blue indifference curve.)

The two green horizontal lines on the graph: they will help us to answer this question.

The top horizontal green line is at 1.75 units of pollution
The bottom horizontal green line is at 1.25 units of pollution

The Chinese guy is currently consuming 1.75 units of pollution and .8 units of goods. He would give up approx. .3 goods (a decrease from .8 to .5 goods) to reduce pollution from 1.75 to 1.25

How much would the Boulder guy pay, in terms of fewer goods, to decrease pollution from 1.75 to 1.25 units? (He is on the black indifference curve.)Approx 2.25 units (a decrease from approx 4.75 goods to approx. 2.5 goods).

For the same change in pollution levels, the Boulder guy will sacrifice 2.25 goods, but the Chinese guy only .3 goods.

Why such a big difference between what they would pay?

Different preferences?

No, we assumed that have the same preferences, the same indifference maps.

The *wtp* for reduced pollution is much higher in Boulder because real income is much higher (the Boulderite starts with a much higher-ranked bundle (on a higher indifference curve).

This result has important environmental implications.

It basically implies that it is more efficient to locate pollution-intensive industries in poor neighorhoods/countries.

This is because if the neighborhood is poor, their *wtp* for less pollution is low relative to the rich neighborhood.

Put simply, poor people relative to rich people, care more about goods than they care about pollution.

If it is a choice between feeding the kids by working in a polluting factory, or no job and hungrey kids, most people would choose the polluting factory, even if would kill them in twenty years. I, on the other hand, would happily have my taxes raised (consume fewer goods) if it substantially reduced global warming.

Many people do not like this result, that, ceteris paribus, efficiency dictates that pollution-intensive industries be located on poor countries/neighborhoods.

Why?

They do not think it is equitable/fair. Its OK that in this case they think equity is more important than efficiency.

A term you often hear is "environmental justice" It is the case that many factories in the U.S. (much heavy industry has already moved abroad) are located in poor neighborhoods, often poor black neighborhoods.

When arguing that it is not fair keep in many that many of the residents would rather have a job and the pollution than no job and a cleaner environment. Also keep in mind that, ceteris paribus, rents are cheaper in dirtier neighborhoods.

Will all this in mind, the equity argument, put simply, is that it is unfair to have an income distribution such that people at one end of the spectrum can afford to live in a clean environment and people at the other end of the spectrum have to live in a dirty environment.

Note that if everyone had the same preferences and the same income, we would all, ceteris paribus, have the same *wtp* for pollution reduction.