1 Notes on congestion and road tolls

Edward Morey Dec. 3, 2018

Some things to think about to get us started.

If road tolls are intented to increase efficiency, road tolls should ideally vary by time of day and day of the week.

If tolls vary by time of day and day of week, they could actually increase the number if drivers on the road and make everyone better off (Henderson 1974). This is an interesting result, most people assume a road toll will reduce the total number of drivers, but it doesn't have to if the toll varies by time of day and day of week.[1]

There is a relationship between speed, flow and the number of cars on the road. This relationship varies by weather (rain, snow, clear) and light (night, dusk, etc.).

2 A stylized road example: Pigou's road to Breck (enridge)

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Assume there are two roads from the front range to Breckenridge: a very wide road and a narrow road, I-70.

Breck is a town and a large ski area.

Assume $10,000$ skiers drive to Breck every Saturday morning (they all have ski passes and feel compelled to go), and they all drive alone (they are paranoid about having others in the car).[2]

Driving time to Breck on the wide road is always 2.5 hours (150 minutes) no matter how many cars are on the wide road - $10,000$ cars would not congest it; it is a wide, magical road.

[1] Note that, in theory, a toll could be negative: e.g subsidize you to drive to Breck if you drive at 3 in the morning.

[2] Assuming a fixed number of trips is unrealistic but simplifies the problem.

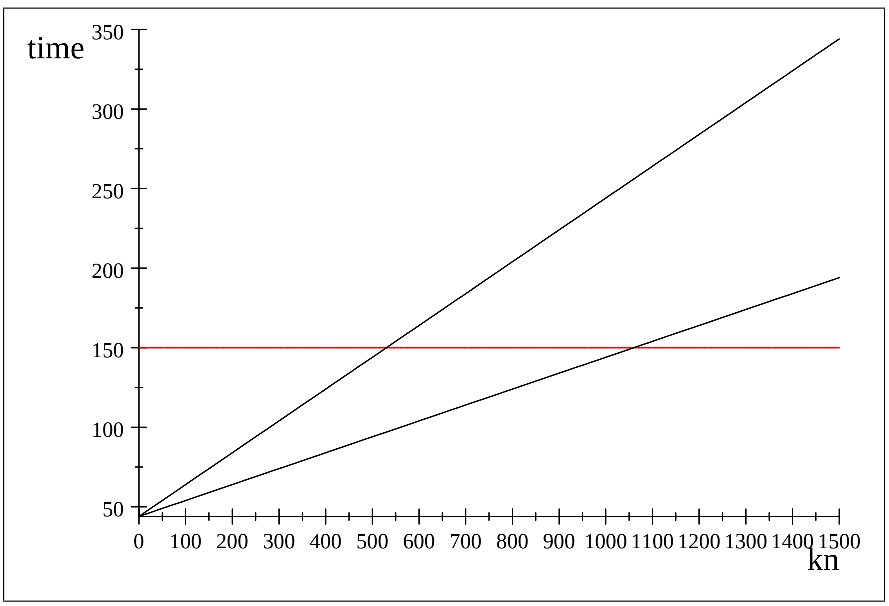
In contrast, the narrow road takes approximately 44 minutes if only one car takes it, but marginal travel time increases as the number of cars on the narrow-road increases (how much total trave time changes), The road is congestible.

Specifically assume $mc_n(k_n) = 44 + .2k_n$ where k_n is the number of cars on the narrow road.[3] For example, if they are 100 cars on the narrow road adding another increases total travel time of the narrow road by $44 + .2(100) = 64$ minutes

Note that by assumption $k_n + k_w = 10,000$. We assumed away the problem of determing the number of people who will want to go to Breck by assuming it fixed at $10,000$.

Given these assumptions total travel time (in min.) for all of the cars on the narrow road is $tc_n(k_n) = 44k_n + .1k_n^2$ and total travel time on the wide road is $tc_w(k_w) = 150k_w$.[4]

It follows that $ac_w = 150$ and $ac_n(k_n) = \frac{44k_n + .1k_n}{k_n} = 44 + .1k_n$. Marginal cost and average cost for the narrow road are the two upward sloping lines. (note that the time it takes you to drive the narrow road is $ac_n(k_n)$, not $mc_n(k_n)$.[5]



[3]The CDOT had a bunch of traffic engineers do a study. They observe average speed with different traffic loads and used the data to estimate this marginal cost function, in terms of time.

[4]How do I get $tc(k_n)$ from $mc(k_n)$? I integrated the marginal cost curve wrt k_n , keeping in mind that $tc_n(0) = 0$.

[5]So, if you are the 100th car on the narrow road it will take everyone on the narrow road $ac_n(100) = 44 + .1(100) = 54$ minutes. But note that total travel time by everyone increases by 64 minutes when the 100th car gets on the road.

Red ac=mc wide road, black narrow (ac lower, mc higher

If everyone has the same value of time, e.g. $10 hour, these costs can also be easily expressed in dollars.[6]

So, what does all of the above say? For the wide road marginal cost equals average cost equals 150 minutes.

For the narrow road, marginal cost is greater than average costs so as the number of cars on the road increases, average driving time increases. For example if, there are currently 1000 cars on the narrow road, marginal cost is 244 minutes (the amount total travel time increases if one more car takes the road), but average cost is only 144 minutes.

Average cost on the narrow road is how long it takes each driver to make it to Breck. If, for example, there are 1000 drivers on the narrow road, the last guy who decides to take the road will take 144 minutes to get to Breck (same time as everyone else) but total travel time goes up not by 144 but rather by 244. Why? The last guy's presence on the road slows everyone else down by .1 minutes (6 seconds)–this is the increase in average time.

[6] This is a strong assumption.

When a driver decides which road to take on Saturday morning, she compares 150 with how long it will take her on the narrow road, which is average cost (how long it will take her). So if

$$\begin{array}{lll} \text{if} & 150 > 44 + .1k_n & \text{take the narrow road} \\ \text{if} & 150 < 44 + .1k_n & \text{take the wide road} \\ \text{if} & 150 = 44 + .1k_n & \text{indifferent} \end{array}$$

So, **in equilibrium** (when no one wants to switch roads), $150 = 44 + .1k_n$,

Solving, $k_n = 1060$. That is, if both roads are common-property resources (access is uncontrolled) , 1060 cars will take the narrow road, and 8940 will take the wide road.

Total travel time for the $10,000$ drivers will be

$$\begin{aligned} tc(1060) &= 44(1060) + .1(1060)^2 + 150(10000 - 1060) \\ &= 1.5 \times 10^6 = 1,500,000 \text{ minutes } = 25,000 \text{ hours } = \$250,000 \end{aligned}$$

Is this the efficient allocation of the $10,000$ cars between the two roads?

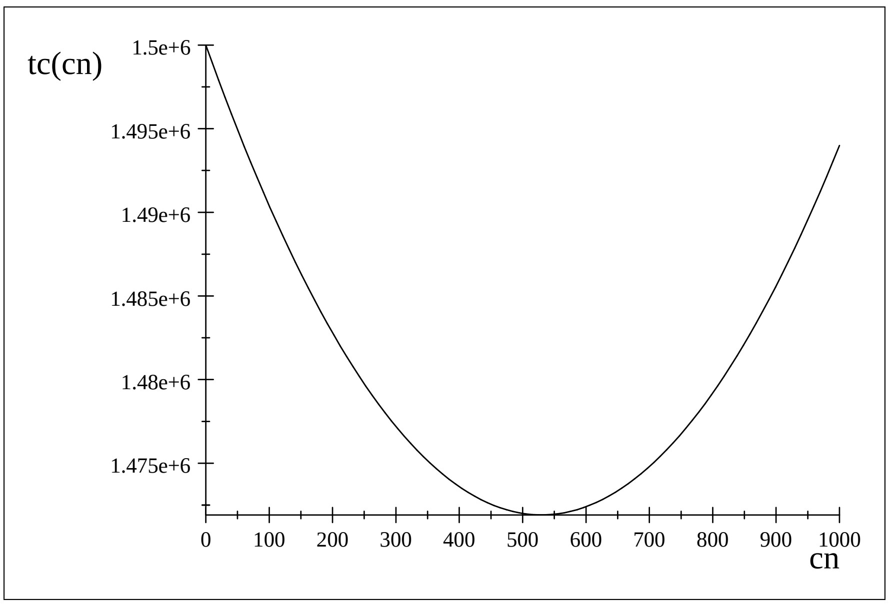
No.

Why not?

Total travel cost is **not** being minimized because there is a wedge between the cost to a driver of taking the narrow road ($ac_n(k_n) = 44 + .1k_n$) and cost to society of her taking the narrow road ($mc_n(k_n) = 44 + .2k_n$). The wedge is $.1k_n$.

What allocation of the cars would minimize total travel time to Breck?. Find the k_n that minimizes

$$tc(k_n) = 44(k_n) + .1(k_n)^2 + 150(10,000 - k_n) = 0.1k_n^2 - 106k_n + 1500\,000$$



How do we find this time minimizing k_n, k_n^*. First find the derivative of $tc(k_n)$ wrt k_n.

$$\begin{aligned}\frac{dtc(k_n)}{dk_n} &= \frac{44(k_n) + .1(k_n)^2 + 150(10000 - k_n)}{dk_n} \\ &= 44 + .2k_n - 150 = 0.2k_n - 106\end{aligned}$$

k_n^* is the k_n for which this derivative is zero. Solving $0.2k_n - 106 = 0$, Solution is: 530.0.

Wow - could this be correct? It says only 530 cars should be on the narrow road and $10000 - 530 = 9470$ on the wide road.

Let's check our answer another way. When the efficient number of cars are on the narrow road the social cost of additional car driving to Breck on the narrow road should equal the social cost of that additional car driving to Breck on the wide road, which is 150 minutes.

Mathematically, marginal cost on narrow road equal marginal cost on wide road when

$$150 = 44 + .2k_n$$

Solution is: 530.0, confirming our answer.

So, how much time is wasted by the misallocation of cars when access is not controlled. Total travel time with the efficient allocation is

$$\begin{aligned} tc(530) &= 44(530) + .1(530)^2 + 150(10000 - 530) \\ &= 1.4719 \times 10^6 = 1471900 \text{ minutes} \\ &= 24,532 \text{ hours } = \$245,320 \end{aligned}$$

The difference is $1.5 \times 10^6 - 1.4719 \times 10^6 = 28100$ minutes $= 28100/60 = 468.33$ hours $= \$4680$ wasted because the cars were inefficiently allocated between the two roads.

How to fix the problem?

Close the narrow road when the 530^{th} car gets on. If this is the solution, 468.33 hours of driving are saved, time that could be used to do other stuff like sleeping or doing one's homework. Some drivers will be made better off.

Will anyone be made worse off (experience increased driving time)? In the common-property equilibrium driving to Breck takes 150 minutes on either road, when the the narrow road is closed after the 530^{th} car, drivers on the wide road still take 150 minutes and those on the narrow road take $44 + .1(530) = 97$ minutes. So, no driver takes more time, and 530 drivers take 53 **minutes** less - definitely a Pareto Improvement.

One problem with this skeme is it might cause a race to the narrow road,I-70. One would want to have a reservation system, register you car online for access, you put down a deposit. You lose your deposit if if you do not show up.

Who will take the the narrow road? And does it matter? Efficiency requires it be the 530 drivers with the highest WTP to save 53 minutes by taking the narrow road (otherwise pareto improvements would remain possible)

But remember we assume the opportunity cost of everyone's time was assumed the same, $\$10/hr$. So, we basically assumed away this issue.[7]

[7] In the real world opportunity cost of time will vary

Or, efficiency could be achieved by charging a toll on the narrow road. What should the toll be? We want to set the toll so that equilibrium is where 530 cars choose to take the narrow road.

First express the toll in minutes, which we will then convert to dollars. In equilibrium, we want marginal **private** costs of taking the narrow road, including the toll ($ac_n() + toll$), to equal 150 when there are 530 cars on the narrow road.

$$150 = 44 + .1(530) + toll_m$$

: Solution is: $toll_m$ equals 53 minutes (we knew this already). That is, the toll should be a wait of 53 minutes where during the wait the driver works for the good of mankind or doesn't wait but pays a dollar toll of $(53/60)10 = \$8.83$.

So are the drivers better off or worse off because we charged the toll. Total cost to the drivers with no toll is $\$250,000$. With the \$8.83 toll it is

$$\begin{aligned} tc &= \$245,320 + \$8.83(530) \\ &= \$250,000 \end{aligned}$$

Wow - the total cost to the 10,000 drivers is the same whether the allocation is efficient or common property. So, the drivers are, not worse off because of the toll, either individually or as a group.[8]

The drivers who paid the toll traded money for time. The money is now available to make others, or even the drivers, better off. It could be used to feed poor kids lunch at school (over a 1000 a week) or even used to pay for improved roads. Of course, if I-70 was widened, the efficient toll would change.

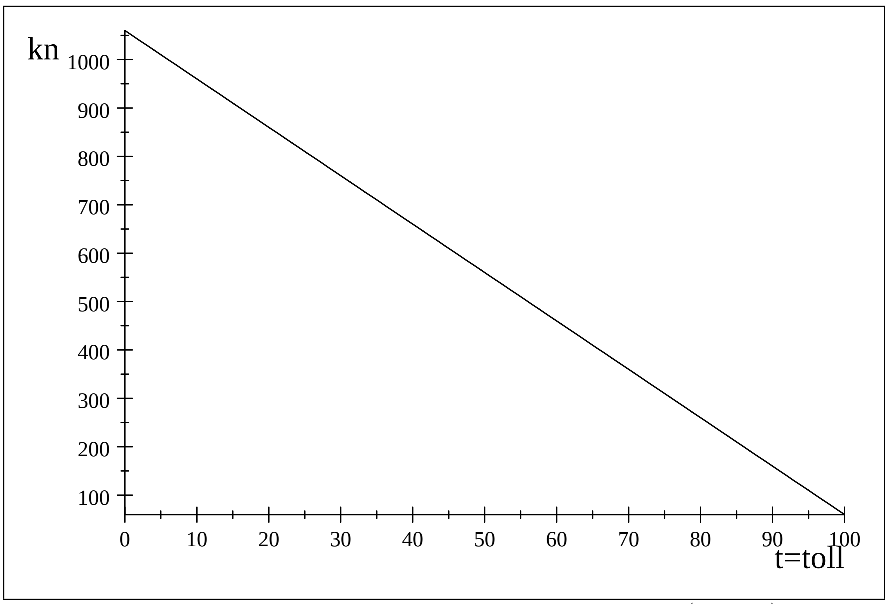
[8] Will this always be the case, or is it just this example?

Now, let's ask another question. What if I-70 was managed to maximize revenues from the toll?

Note that if there are no maintence costs that vary with the number of cars (a simplifying assum.), maximizing revenues will max profits.

First we need to determine the number of cars that will take the narrow road as a function of the toll.

We know in equilibrium, $150 = 44 + .1(k_n) + toll_m$; solving $k_n = k_n(toll_m) = 1060.0 - 10.0 toll_m$, which is the demand function for trips on the narrow road.



Cars on the narrow road as a function of the toll (in min)

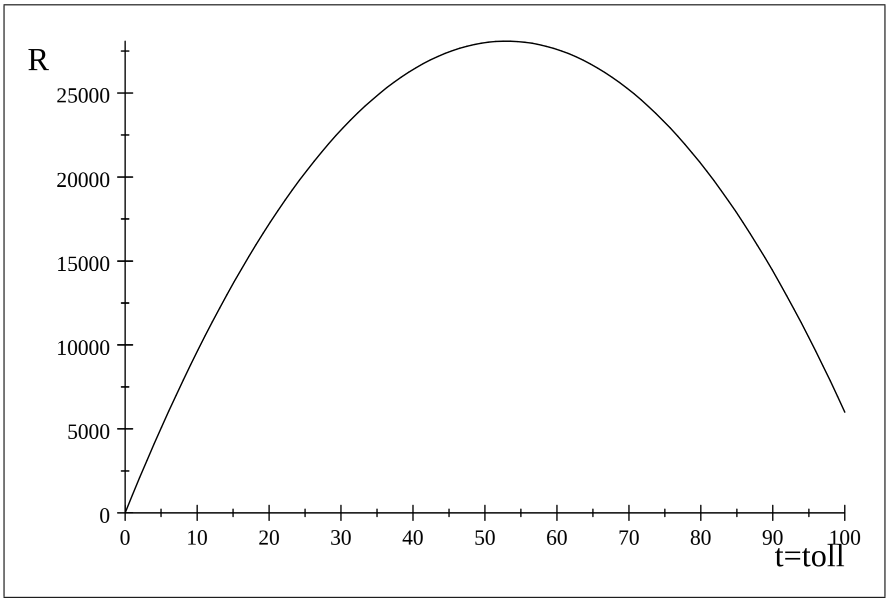
Note that demand is 1060 if the toll is zero.

So, revenue from the toll is toll times demand as a function of the toll[9]

$$R(t) = (1060.0 - 10.0t)t = 1060t - 10t^2$$

At what t i s the revenue from the toll maximized. Graphing it $1060t - 10t^2$

[9] For brevity let t denote *toll*.



Notice where this is maximized, at 53 minutes, the same answer we got when we choose the toll to minimize total travel time by the $10,000$ cars.

Wow - a private owner maximizing revenues[10] would achieve the efficient allocation of cars between the two roads. Adam Smith's invisible cruise control

[10] maximizing revenues would maximize profits if maintenance was not a function of the number of cars

This road example is obviously stylized and restrictive - but it gives the flavor of things. Many road are highly congested from an efficiency point of view, but many are not.

Restrictive assumptions included the existence of a non-congestible substitute and the assumption that the number of Saturday morning skiers would **not** be affected by the toll or access restriction: a toll that varied by time of day and day of week would cause substitution away from driving I-70 on Saturday morning, substitution to other days and times, and substitution to sleeping and watching football. Also my restrictive assumption that everyone had the same value of time.

The writings of Toby Page motivated these notes.

Note that if there are two congestible roads, it is unlikely that efficiency could be achieved by putting a toll on only one of them. (The untolled road will remain too congested, probably more congested than it had been)

Note that tolls, or their lack, affect where people choose to live, and where firms choose to locate.

Should the ski area like, or dislike, a variable toll? How about towns like Evergreen and Silverthorn?

https://en.wikipedia.org/wiki/London_congestion_charge

The London congestion charge is a fee charged on most motor vehicles operating within the **Congestion Charge Zone** (CCZ)[1] in Central London between 07:00 and 18:00 Mondays to Fridays.[2] It is not charged on weekends, public holidays or between Christmas Day and New Year's Day (inclusive).[3] The charge was introduced on 17 February 2003. As of 2017, the London charge zone remains as one of the largest congestion charge zones in the world, despite the cancellation of the Western Extension which operated between February 2007 and January 2011. T**he charge aims to reduce high traffic flow and pollution in the central area and raise investment funds** for London's transport system.

The standard charge is **£11.50 for each day, for each non-exempt vehicle** that travels within the zone, with a penalty of between £65 and £195 levied for non-payment. In July 2013 the Ultra Low Emission Discount (ULED) introduced more stringent emission standards that limit the free access to the congestion charge zone to all-electric cars, some plug-in hybrids, and any vehicle that emits 75g/km or less of CO2 and meets the Euro 5 standards for air quality. The ULED scheme was designed to curb the growing number of diesel vehicles on London's roads, which since June 2016 pay the full congestion charge.[4][5] **The T-charge (toxity charge) was introduced from October 2017 for vehicles that do not meet Euro 4 standards. These older polluting vehicles pay an extra £10 charge on top of the congestion charge to drive within the Congestion Charge Zone**.[6][7][8] From April 2019, the T-charge will be replaced by the Ultra-Low Emission Zone, which will apply to vehicles which do not meet Euro 5 standards and operate 24/7. From 2021, the ULEZ will be extended to the North and South Circular.[9]

Enforcement is primarily based on automatic number plate recognition (ANPR). Transport for London (TfL) is responsible for the charge which has been operated by IBM since 2009. During the first ten years since the introduction of the scheme, gross revenue reached about £2.6 billion up to the end of December 2013. From 2003 to 2013, about £1.2 billion (46%) of net revenue has been invested in public transport, road and bridge improvement and walking and cycling schemes. Of these, a total of £960 million was invested on improvements to the bus network.

In 2013, ten years after its implementation in 2003, **TfL reported that the congestion charging scheme resulted in a 10% reduction in traffic volumes from baseline conditions, and an overall reduction of 11% in vehicle kilometres in London between 2000 and 2012.** Despite these gains, traffic speeds have also been getting progressively slower over the past decade, particularly in central London. TfL explains that the historic decline in traffic speeds is most likely due to interventions that have reduced the effective capacity of the road network to improve the urban environment, increase road safety and prioritise public transport, pedestrian and cycle traffic, as well as an increase in road works by utilities and general development activity since 2006. TfL concludes that while levels of congestion in central London are close to pre-charging levels, the effectiveness of the congestion charge in reducing traffic volumes means that conditions would be worse without the Congestion Charging scheme.[10]

Consider the following data from

I-70 Mountain Corridor:Progammatic Environmental Impact Statement - Technical Advisory Committee, June 2000. I think it was prepared by J.F. Sato and Associates in Littleton.

It indicates that Westbond I-70 on Saturdays in the winter at Idaho Springs has a volume of almost 4000 cars per hour at 7 a.m., official capacity is about 3000 cars per hour. Eastbound reaches its peak on Saturday and Sunday afternoon at around 4 p.m. with a volume of about 3500 cars per hour.

Joy
Thank you so much.
I will send you what I write up, when I finish it.
Thanks again.
Edward

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From: Joy, Cecelia [mailto:Cecelia.Joy@dot.state.co.us]
Sent: Monday, October 17, 2005 4:37 PM
To: Edward Morey
Cc: Paulsen, Chris
Subject: RE: I-70 corridor - research question.

Ed, I'm not sure of the "June 200?) PEIS report that you reference. The most current source of information is contained in the I-70 PEIS, Vol 1 and 2 dated December 04. The data you are interested in is provided in Appendix b of Volume 2. See www.i70mtncorridor.com. Let me or Chris Paulsen (the project manager) know if you have any other questions.

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From: Edward Morey [mailto:Edward.Morey@Colorado.edu]
Sent: Sunday, October 16, 2005 4:25 PM
To: Joy, Cecelia
Subject: I-70 corridor - research question.

Joy Cecelia

I am an economist at C.U. who has an interest in transportation economics.

I am wondering if there is a study or estimate of the relationship between travel times and traffic volume on I-70 west of Denver. For example, how long it takes to drive from Golden to the tunnel as a function of traffic volume

I am particularly interested in winter travel between Denver and the tunnel, but anything related would be great. Thanks.

Any information you might have would be greatly appreciated.

I already have a copy of the June 200 PEIS report

Thanks

Edward

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