

## Defining the compensating variation, $cv$ , and the equivalent variation, $ev$ .

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More complicated stuff, but important stuff for you to understand.

Remember our travel-cost discussion about the CS associated with a change in the quality of the South Fork. How the demand curve for trips to the North Fork would shift in if the South Fork was cleaned up. That was a window into why the area under a demand function above a price/cost might not be a good measure of WTP or WTA (these are notes we might, or might not have yet covered)

It is actually even more complicated: one cannot accurately identify WTP or WTA as the sum of areas under even a bunch of demand functions.

My TC example with the North and South Forks enlightened somewhat, but also mislead.

Consider the inverse demand curves for Diet Coke and Diet Pepsi (assuming each has a price and then the price of Coke rises. The problem is that when the price of Coke changes this causes the demand curve for Pepsi to shift, and this causes the price of Pepsi to change, which then causes the demand curve for Coke to shift. And on and on. It turns out that the areas under demand functions is not well defined, so not a well-defined measure of a money measure of a utility change.

So a better **money** measure of an individual's utility change needs to be found.

What properties would one like this measure to have?

1. Positive for improvements (policies that cause the individual's WB/utility to increase)
2. Negative for deteriorations (policies that cause the individual's WB/utility to decrease)
3. The money measure should be an adjustment to your income such that after the adjustment you are indifferent between the original state (without the policy in place) and the new state (with the policy in place)
4. The money measure is well defined for both improvements and deteriorations.

Two of the money asures that fit the bill are the compensating variation,  $cv$ , and the equivalent variation,  $ev$ .

They differ in terms of whether the income adjustment occurs in the original state or the new state.

Simple definitions of the compensating and equivalent variation

## 1 The $cv$ and $ev$ are both money measures of of a WB change

The  $cv_i$  and  $ev_i$  are calculated for individual  $i$  for a change in the state of the world.

### 1.1 States of the world

A *state-of-the-world*, SOW, is all that influences your life that is exogenous to you.

What do we mean by a change in the state of the world? We will define a state of the world from an individual's perspective. Let  $S_i^0$  denote the initial state of the world as experienced by individual  $i$ , and let  $S_i^1$  denote the new/proposed state. This is just useful notation.

We want to determine individual  $i$ 's monetary value for a change from  $S_i^0$  to  $S_i^1$ .

The question is how to describe/characterize a state of the world for individual  $i$ ?

A state of the world is defined in terms of that which is exogenous to the individual (prices, levels of public goods, environmental quality, etc.)

For example, Lady Gaga is one of the characteristics of the state you live in. So, are the prices of coke and Coke, and the fact that the FlatIrons are placed where they are placed, and that, at the moment, your significant other is probably not named Wanda Sue.



Now let's introduce some notation for states of the world

Looking at things simply, the initial state for individual  $i$  might be completely described by the following vector<sup>1</sup>

$$\mathbf{S}_i^0 \equiv (y_i^0, \mathbf{P}^0, \mathbf{C}^0)$$

where  $y_i^0$  is individual  $i$ 's income in the initial state,

$\mathbf{P}^0 \equiv [p_1^0, p_2^0, \dots, p_M^0]$  is a the price vector for market goods in the initial state

$\mathbf{C}^0 \equiv [c_1^0, c_2^0, \dots, c_J^0]$  is a vector of the levels of the nonmarket commodities in the initial state (including the existence of Ms. Gaga)

For example,  $p_4^0$  is the price of market good 4 in the initial state and  $c_7^0$  is the level of non-market commodity 7 in the initial state.

A state of the world includes everything that affect the quality of your life, but whose levels you cannot control.<sup>2</sup>

The proposed state for individual  $i$  is completely described by the levels of exogenous variables that would exist in the proposed state; e.g.  $\mathbf{S}_i^1 \equiv \{y_i^1, \mathbf{P}^1, \mathbf{C}^1\}$ .

For example, if all that changed between the two states is the levels of some of the non-market commodities the change would be from  $\{y_i^0, \mathbf{P}^0, \mathbf{C}^0\}$  to  $\{y_i^0, \mathbf{P}^0, \mathbf{C}^1\}$ .

We are interested determining the value an individual places on a change in the state of the world. This value can be positive or negative.

For example, if we enact policies to reduce global warming, the rate of global warming will be reduced, the price of carbon energy will increase, the price of products that are carbon intensive will increase, your income might go up or down, etc. How would you as individual  $i$  value such a change in the state of the world.

Do you prefer the initial state or this new state with a reduction in GW and all of the associated effects?

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<sup>1</sup>It could include a lot of other things as well such as the incomes of other people.

<sup>2</sup>Note that some levels of some non-market commodities might be individual specific, but I have assumed that away. Give this footnote more thought: One aspect of the word for everyone is that Edward's mom was named Esther. Obviously that might be more important for me than for you.

The introduction of the City of Boulder's Open-space Program many years ago affected Boulder in many ways (land preservations, housing prices, incomes, congestion, growth patterns in neighboring communities, etc.). Note that the program has affected many people over the years, many of whom have never lived in Boulder. How would one value such a change in the state of the world?

## 2 One measure of the value of a state change is the compensating variation, another is the equivalent variation

The  $cv$  and the  $ev$  are the money measures of welfare changes (the consumer's surplus measures) that we want to estimate and want to use, **we don't want to use** the area under the inverse demand function of a particular commodity.

Individual  $i$ 's  $cv$  for a change from  $S^0$  to  $S^1$ ,  $cv_i$ , is how much money would have to be **subtracted** from her income in the **new** state to make her indifferent between the initial state and the new state with the subtraction from income. Formally the  $cv$  is defined as

$$\{y_i^0, \mathbf{P}^0, \mathbf{C}^0\} \sim \{(y_i^1 - cv_i), \mathbf{P}^1, \mathbf{C}^1\}.$$

where  $\sim$  denotes indifference.

$cv > 0$  for improvements and  $cv < 0$  for deteriorations.

For an **improvement**,  $cv$  is willingness to pay,  $WTP$ , for the new state (how much she would pay in the new state to be in the new state).  $cv > 0$  for improvement and negative for deteriorations. For a **deterioration**,  $cv$  is, in absolute terms, willingness to accept,  $WTA$ , the new state (how much she would have to be paid in the new state to accept it).

Individual  $i$ 's  $ev$  for a change from  $S^0$  to  $S^1$ ,  $ev_i$ , is how much money would have to be **added** to income in the **initial** state to make her indifferent between the new state and the initial state with the income addition

$$\{(y_i^0 + ev_i), \mathbf{P}^0, \mathbf{C}^0\} \sim \{y_i^1, \mathbf{P}^1, \mathbf{C}^1\}.$$

$ev > 0$  for improvements and  $ev < 0$  for deteriorations. For an **improvement**,  $ev$  is  $WTA$  the initial state (how much she would have to be paid in the initial state to forego the improvement). For a **deterioration**,  $ev$  is, in absolute terms,  $WTP$  to not switch to the new state (how much she would pay in the initial state to not switch).

One can show that  $cv \leq ev$  for both improvements and deteriorations. Note that  $cv$  and  $ev$  are negative numbers for deteriorations, and  $-5 < -4$ .

For a price or quality change, consumer's surplus as measured by the area (or change in area) under an inverse demand function is typically neither the compensating variation nor the equivalent variation. It approximates them both. Sometimes the approximation is a good approximation, sometimes it is not.

### 3 More complicated but fun to think about - include the characteristics of market goods as an aspect of the state of the world

#### Or, How to make a fortune in marketing research

Individuals also take as exogenous the characteristics of market goods and the above notation can be generalized to incorporate this.

Define the  $cv$  and  $ev$  associated with a change in the characteristics of a market good. For example, one characteristic of a Diet Coke is that it has zero calories—the consumer gets to choose the number of cans of Diet Coke he will consume but not how many calories are in each can.

Many people in marketing try to estimate the  $cv$  or  $ev$  associated with a change in the characteristics of a good. For example, what is your WTP for adding GPS to a car, or decreasing the probability that the drugs you consume are tainted with a chemical that might make you crazy (this was a significant problem when I was in college; e.g. coke laced with speed).

Think of a market good as a package/vector of characteristic levels where  $a_{jk}$  is the amount of characteristic  $k$  in market good  $j$ . For example,  $k$  might be size or level of sweetness. The vector  $\mathbf{a}_j \equiv (a_{j1}, a_{j2}, \dots, a_{jK})$  is then a complete description of market good  $j$ . And  $\mathbf{A} \equiv (\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_M)$  is a complete description of all  $M$  market goods.

Recognizing this,  $\mathbf{S}$  becomes  $\mathbf{S}_i^s \equiv (y_i^s, \mathbf{P}^s, \mathbf{C}^s, \mathbf{A}^s)$ ,  $s = 0, 1$ . With this generalization, The  $cv$  is the amount of money such that

$$(y_i^0, \mathbf{P}^0, \mathbf{C}^0, \mathbf{A}^0) \sim ((y_i^1 - cv), \mathbf{P}^1, \mathbf{C}^1, \mathbf{A}^1)$$

and the  $ev$  is

$$((y_i^0 + ev), \mathbf{P}^0, \mathbf{C}^0, \mathbf{A}^0) \sim (y_i^1, \mathbf{P}^1, \mathbf{C}^1, \mathbf{A}^1)$$

In terms of this more general characterization of states of the world, we can define an individual's  $cv$  for a decrease in the number of calories in a regular coke, holding all of its other characteristics constant, or, for a calorie decrease combined with a price increase.

Marketing research spends millions of dollars trying to estimate such consumer's surplus values. (There are many similarities between what environmental economists do and what market researchers do.) Edward, for example, has a coauthor that is a market researcher.



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Thinks about also including in the state of the world, stuff like friends and significant others. E.g.  $\mathbf{F}_i^s$  might be individual  $i$ 's friends set in state  $s$ .