A Simple Improvement

This section was deleted from the primer paper.

Briefly consider a price decrease in alternative 1, $p_1^1 < p_1^o$. Depending on the ε draw, an individual will fall into one of four groups:

- Group A: Individuals who do not choose alternative 1 either before or after its price decreases.
- Group B: Individuals that choose alternative 1 both before and after its price decreases.
- Group C: Individuals who switch from alternative 2 to alternative 1
- Group D: Individuals who switch from alternative 3 to alternative 1 (note that no one switches to alternatives 2 or 3)

Note that the switch groups are grouped in terms of the alternative the individual switches away from.

For individuals in group A, $cv = 0$ and $m = y^o$. For individuals in group B, $cv > 0$ and equal to $p_1^o - p_1^1$. If in group B, the expenditure level required to keep the individual at his original utility level, u^o, is $y^o + (p_1^1 - p_1^o) = \mu < y^o$. If an individual is in group C or D, his cv is between 0 and $(p_1^o - p_1^1) > 0$. In terms of the required levels of expenditures, an individual in group C or D will require expenditures less than y^o and greater than μ.

As with the deterioration, our expectation of the level of expenditures required to make an individual whole can be decomposed into a number of terms

$$E[m] = c_A + c_B + c_C + c_D$$

All individuals in group A require the same expenditure level, y^0, to make them whole in the new state, so,

$$c_A = \Pr(\text{in } A : y^o) y^o$$

where

$$\Pr(\text{in } A : y^o) = 1 - P(1 : y^o, y^o, y^o, p_1^1, p_2^o, p_3^o) \qquad \#$$

Likewise, all individuals in group B require the same expenditure level, μ, to make them whole in the new state, so

$$\begin{aligned} c_B &= \Pr(\text{in } B\text{: } \mu)\mu \\ &= P(1 : \mu, y^o, y^o, p_1^1, p_2^o, p_3^o) \end{aligned}$$

If the individual chooses an alternative at the old price, he will continue to choose the alternative after the price has decreased.

For group C,

$$c_C = -\int_{>\mu}^{<y^o} m \frac{\partial P(2 : m, y^o, y^o, p_1^1, p_2^o, p_3^o)}{\partial m} dm \qquad \#$$

where $P(2 : m, y^o, y^o, p_1^1, p_2^o, p_3^o)$ is the probability of the alternative individuals in group C switch away from, and m is the expenditure level associated with the alternative they will switch to (alternative 1). Those in group C choose alternative 1 after its price has decreased, and alternative 2 before, so the relevant prices are p_1^1, p_2^o and p_3^o. As m increases in the range μ to y^o, $P(2 : m, y^o, y^o, p_1^1, p_2^o, p_3^o)$ decreases; that is, as the expenditure level associated with choosing alternative 1 increases, holding constant at y^o the expenditure level associated with alternatives 2 and 3, it becomes more likely that the individual will switch to alternative 1 (and abandon alternative 2).

Finally, for group D,

$$c_D = -\int_{>\mu}^{<y^o} m \frac{\partial P(3 : m, y^o, y^o, p_1^1, p_2^o, p_3^o)}{\partial m} dm \qquad \#$$

where $P(3 : m, y^o, y^o, p_1^1, p_2^o, p_3^o)$ is the probability of the alternative individuals in group D switch away from, and m is the expenditure level associated with the alternative they will switch to (alternative 1).

Consider a numerical example where, $\beta = .1$, $y^o = 100$, $p_1^o = 97$, $p_2^o = 95$, $p_3^o = 96$ and $p_1^1 = 95$. In which case,

$$\begin{aligned}\Pr(\text{in } A : 100) &= 1 - P(1 : 100, 100, 100, 95, 95, 96) \qquad \# \\ &= 1 - \frac{\exp(.1(100-95)^2)}{\exp(.1(100-95)^2) + \exp(.1(100-95)^2) + \exp(.1(100-96)^2)} \\ &= 0.58447\end{aligned}$$

So, $c_A = \Pr(\text{in } A\text{: } y^o) y^o = (0.58447)100 = \$58.447$.

For this price decrease, $y^o + (p_1^1 - p_1^o) = \mu = 100 - 2 = 98$. so

$$\begin{aligned}\Pr(\text{in } B : 98) &= P(1 : 98, 100, 100, 95, 95, 96) \\ &= \frac{\exp(.1(98-95)^2)}{\exp(.1(98-95)^2) + \exp(.1(100-95)^2) + \exp(.1(100-96)^2)} \\ &= 0.12552 \qquad \#\end{aligned}$$

and $c_B = \Pr(inB : 100)98 = (0.12552)98 = \$12.301$.

For group C,

$$\begin{aligned}c_C &= -\int_{98}^{100} m \frac{\partial}{\partial m}\left(\frac{\exp(.1(100-95)^2)}{\exp(.1(\mathrm{m}-95)^2) + \exp(.1(100-95)^2) + \exp(.1(100-96)^2)}\right) dm \\ &= \$20.455 \qquad \#\end{aligned}$$

This calculation was done using a call to *Maple* in *Scientific Workplace,* as was the next calculation

$$\begin{aligned}c_D &= -\int_{98}^{100} m \frac{\partial}{\partial m}\left(\frac{\exp(.1(100-96)^2)}{\exp(.1(\mathrm{m}-95)^2) + \exp(.1(100-95)^2) + \exp(.1(100-96)^2)}\right) dm \\ &= \$8.3166 \qquad \#\end{aligned}$$

Concluding,

$$E[cv] = y^0 - E[m] = 100 - (58.447 + 12.301 + 20.455 + 8.3166) = \$0.4804 > 0 \qquad \#$$

which is positive, as required, but closer to zero than to $2 because 58% have a *cv* of zero and only 12.5% have a *cv* of $2.