



## External Economies and Diseconomies in a Competitive Situation

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## EXTERNAL ECONOMIES AND DISECONOMIES IN A COMPETITIVE SITUATION<sup>1</sup>

### I. THE SCOPE OF THE PAPER

THE purpose of this note is to distinguish between certain types of external economies and diseconomies which are connected with marginal adjustments in purely competitive situations. We shall not be dealing with divergences between private and social interests due to monopolistic or monopsonistic situations, nor with any of the problems which arise from indivisibilities such as the lumpiness of investment in particular forms, nor with any questions about large structural changes such as whether a particular industry should exist at all or not. We shall be concerned only with small adjustments to existing competitive situations.

### II. THE COMPETITIVE SITUATION WITH NO EXTERNAL ECONOMIES OR DISECONOMIES

Let us consider two industries. These "industries" may or may not in fact produce identically the same product and so in reality constitute a single industry. That is immaterial to our general theory. But we assume that within each "industry" there are a large number of independent competing firms, so that to each individual entrepreneur the price of the product and of the factors is given. In the absence of any external economies or diseconomies, each entrepreneur will hire each factor up to the point at which the additional product of the factor multiplied by its price is equal to the price of the factor. Moreover, there will be constant returns to scale. If every factor in either of our two industries were increased by 10%, including the number of entrepreneurs, then the product also would be increased by 10%.

Let us write  $x_1$  and  $x_2$  for the products of industry 1 and industry 2 respectively. We assume that there are two factors,  $l$  and  $c$ , or labour and capital, employed in both industries, so that  $l_1 + l_2 = l$  and  $c_1 + c_2 = c$ . We will write  $\bar{x}_1, \bar{l}_1, \bar{c}_1$ , etc., for the market prices of the products and factors; and  $X_1 = x_1\bar{x}_1, L_1 = l_1\bar{l}_1, C_1 = c_1\bar{c}_1$ , etc., for the total value of the output of

<sup>1</sup> This note has arisen, out of a consideration of the problems of the economic development of under-developed territories, in the preparation of Volume II of my *Theory of International Economic Policy* for the Royal Institute of International Affairs.

$x_1$  or for the total income earned by  $l_1$ , etc. Finally, we shall write  $\bar{L}_1, \bar{C}_1$ , etc., for the amounts which the factors would have to be paid if they received the value of their marginal social net products. In our model capital is always the hiring factor, and its reward is, therefore, always equal in each industry to the total output of that industry minus the wages paid to labour in that industry, so that  $C_1 = X_1 - L_1$  and  $C_2 = X_2 - L_2$ .

In the case in which there are no divergences between private and social net products we can write

$$\left. \begin{aligned} x_1 &= H_1(l_1, c_1) \\ x_2 &= H_2(l_2, c_2) \end{aligned} \right\} \dots \dots \dots (1)$$

where  $H_1$  and  $H_2$  are homogeneous functions of the first degree, expressing the fact that there are constant returns to scale in both industries. Now

$$x_1 = \frac{\partial x_1}{\partial l_1} l_1 + \frac{\partial x_1}{\partial c_1} c_1$$

or 
$$1 = \frac{l_1}{x_1} \frac{\partial x_1}{\partial l_1} + \frac{c_1}{x_1} \cdot \frac{\partial x_1}{\partial c_1}$$

We shall write  $\epsilon_{l_1}^{x_1}$  for  $\frac{l_1}{x_1} \cdot \frac{\partial x_1}{\partial l_1}$  and so on, so that we have

$$\epsilon_{l_1}^{x_1} + \epsilon_{c_1}^{x_1} = \epsilon_{l_1}^{x_2} + \epsilon_{c_2}^{x_2} = 1 \dots \dots (2)$$

These expressions describe the fact that if, for example, a 10% increase in labour alone causes a 3% increase in output, then a 10% increase in capital alone must cause a 7% increase in output, because a 10% increase in both factors will cause a 10% increase in output.

In this situation  $l_1$  will be paid a money wage ( $L_1$ ) equal to  $\frac{\partial x_1}{\partial l_1} l_1 \bar{x}_1$  or  $\epsilon_{l_1}^{x_1} X_1$ , and this will also be equal to the value of its marginal social net product. Capital in industry 1 will receive  $X_1 - L_1$  which from equation (2) equals  $\epsilon_{c_1}^{x_1} X_1$ , which is also equal to the value of capital's marginal social net product, so that in this case we have

$$\begin{aligned} L_1 &= \bar{L}_1 = \epsilon_{l_1}^{x_1} X_1 & L_2 &= \bar{L}_2 = \epsilon_{l_2}^{x_2} X_2 \\ C_1 &= \bar{C}_1 = \epsilon_{c_1}^{x_1} X_1 & C_2 &= \bar{C}_2 = \epsilon_{c_2}^{x_2} X_2 \end{aligned}$$

Moreover, since  $\epsilon_{l_1}^{x_1} = \frac{L_1}{X_1}$ , we can measure  $\epsilon_{l_1}^{x_1}$  from the proportion of the total product in industry, which goes to labour. And similarly for the measurement of  $\epsilon_{c_1}^{x_1}$ ,  $\epsilon_{l_2}^{x_2}$  and  $\epsilon_{c_2}^{x_2}$ .

### III. TWO TYPES OF EXTERNAL ECONOMY AND DISECONOMY

Such is the simplest competitive model. We intend now to consider cases where what is done in one industry reacts upon the conditions of production in the other industry in some way other than through the possible effect upon the prices of the product or of the factors in that other industry. All such reactions we shall describe as constituting external economies or diseconomies, because the individual entrepreneur in the first industry will take account of the effect of his actions only upon what happens inside the first industry (the internal effect), but will leave out of account the effect of his actions upon the output of the second industry, in which it may improve production (an external economy) or diminish production (an external diseconomy).

But the purpose of this note is to distinguish between two types of such external economies or diseconomies. The first type we shall call "unpaid factors of production," and the second the "creation of atmosphere." The essential difference between these two types of external economy or diseconomy is that in the first case there are still constant returns to scale for society as a whole, though not for the individual industry, whereas in the second case there are still constant returns to scale for each individual industry but not for society as a whole.

### IV. UNPAID FACTORS

Suppose that in a given region there is a certain amount of apple-growing and a certain amount of bee-keeping and that the bees feed on the apple-blossom. If the apple-farmers apply 10% more labour, land and capital to apple-farming they will increase the output of apples by 10%; but they will also provide more food for the bees. On the other hand, the bee-keepers will not increase the output of honey by 10% by increasing the amount of land, labour and capital applied to bee-keeping by 10% unless at the same time the apple-farmers also increase their output and so the food of the bees by 10%. Thus there are constant returns to scale for both industries taken together: if the amount of labour and of capital employed both in apple-farming and bee-keeping are doubled, the output of both apples and honey will be doubled. But if the amount of labour and capital are doubled in bee-keeping alone, the output of honey will be less than doubled; whereas, if the amounts of labour and capital in apple-farming are doubled, the output of apples will be doubled and, in addition, some contribution will be made to the output of honey.

We call this a case of an unpaid factor, because the situation is due simply and solely to the fact that the apple-farmer cannot charge the bee-keeper for the bees' food, which the former produces for the latter. If social-accounting institutions were such that this charge could be made, then every factor would, as in other competitive situations, earn the value of its marginal social net product. But as it is, the apple-farmer provides to the bee-keeper some of his factors free of charge. The apple-farmer is paid less than the value of his marginal social net product, and the bee-keeper receives more than the value of his marginal social net product.

This situation is shown if industry 1 represents bee-keeping and industry 2 apple-farming and if we replace equations (1) and (2) with

$$\left. \begin{aligned} x_1 &= H_1(l_1, c_1, x_2) \\ x_2 &= H_2(l_2, c_2) \end{aligned} \right\} \dots \dots (4)$$

so that  $\epsilon_{l_1}^{x_1} + \epsilon_{c_1}^{x_1} + \epsilon_{x_2}^{x_1} = \epsilon_{l_1}^{x_2} + \epsilon_{c_2}^{x_2} = 1$

In this case  $l_1$  will be paid the value of its marginal social net product, and we have  $L_1 = \bar{L}_1 = \epsilon_{l_1}^{x_1} X_1$ .  $c_1$  will be paid  $X_1 - L_1$  or  $\epsilon_{c_1}^{x_1} X_1 + \epsilon_{x_2}^{x_1} X_2$ ; but  $\epsilon_{c_1}^{x_1} X_1$  is the value of  $c_1$ 's marginal social net product, so that we have  $C_1 = \bar{C}_1 + \epsilon_{x_2}^{x_1} X_1$ . In other words,  $c_1$  will have to have its earnings taxed at an *ad valorem* rate of  $\frac{X_1}{C_1} \epsilon_{x_2}^{x_1}$  in order to be paid a net reward equal to the value of its marginal social net product.

But, on the other hand,  $l_2$  and  $c_2$  will be paid just so much less than the value of their marginal social net products.

$$\begin{aligned} \bar{L}_2 &= l_2 \left( \bar{x}_2 \frac{\partial x_2}{\partial l_2} + \bar{x}_1 \frac{\partial x_1}{\partial x_2} \cdot \frac{\partial x_2}{\partial l_2} \right) \\ &= \epsilon_{l_2}^{x_2} X_2 \left( 1 + \frac{X_1}{X_2} \epsilon_{x_2}^{x_1} \right) \end{aligned}$$

But  $l_2$  will receive only  $\epsilon_{l_2}^{x_2} X_2$ , so that  $\bar{L}_2 = L_2 \left( 1 + \frac{X_1}{X_2} \epsilon_{x_2}^{x_1} \right)$  and the wages of labour in apple-farming will need to be subsidised at an *ad valorem* rate of  $\frac{X_1}{X_2} \epsilon_{x_2}^{x_1}$  in order to equate rewards to the value of the factor's marginal social net product. Similarly,  $\bar{C}_2 = C_2 \left( 1 + \frac{X_1}{X_2} \epsilon_{x_2}^{x_1} \right)$ , and the same *ad valorem* rate of subsidy should be paid to the earnings of capital in apple-farming. Since

$C_2 + L_2 = X_2$ , the total tax revenue of  $X_1 \epsilon_{x_1}^{x_1}$  raised on  $C_1$  will be equal to the two subsidies of  $C_2 \frac{X_1}{X_2} \epsilon_{x_2}^{x_1}$  and  $L_2 \frac{X_1}{X_2} \epsilon_{x_2}^{x_1}$ .<sup>1</sup>

In order to discover the appropriate rates of tax and subsidy the essential factor which will need to be estimated is  $\epsilon_{x_1}^{x_1}$ , the percentage effect on the output of honey which a 1% increase in the output of apples would exercise.

Now the relationship which we have just examined might be a reciprocal one. While the apples may provide the food of the bees, the bees may fertilise the apples.<sup>2</sup> Once again we may have constant returns to scale for society as a whole; a 10% increase in all factors in both industries would cause a 10% increase in the output of both products. In this case instead of equations (4) we should have

$$\left. \begin{aligned} x_1 &= H_1(l_1, c_1, x_2) \\ x_2 &= H_2(l_2, c_2, x_1) \end{aligned} \right\} \dots (5)$$

$$\epsilon_{l_1}^{x_1} + \epsilon_{c_1}^{x_1} + \epsilon_{x_2}^{x_1} = \epsilon_{l_2}^{x_2} + \epsilon_{c_2}^{x_2} + \epsilon_{x_1}^{x_2} = 1$$

By a process similar to that adopted in the previous case we can obtain formulæ to show what subsidies and taxes must be imposed in order to equate each factor's income in each industry to the value of its marginal social net product.

We can obtain the actual rewards of the factors in exactly the same way as in the previous example. Labour in industry 1 will obtain a wage equal to the value of its marginal private net product or  $\bar{x}_1 \frac{\partial x_1}{\partial l_1}$ , so that  $L_1 = \epsilon_{l_1}^{x_1} X_1$ . Capital in industry 1 will receive the remainder, or  $X_1 - L_1$ , so that from equations (5)  $C_1 = X_1(\epsilon_{c_1}^{x_1} + \epsilon_{x_2}^{x_1})$ . Similarly,  $L_2 = \epsilon_{l_2}^{x_2} X_2$  and  $C_2 = X_2(\epsilon_{c_2}^{x_2} + \epsilon_{x_1}^{x_2})$ .

To obtain expressions for the value of each factor's marginal social net product we have now to allow for the repercussions of each industry upon the other. Thus the value of the marginal social net product of labour in apple-farming includes not only the increased output of apples directly produced but also the increased output of honey caused by this increase in apple-output plus the further increase in apple-output due to this increase in honey-output plus the still further increase in honey-output due to this

<sup>1</sup> In this case it would, of course, have exactly the same effect if the subsidy were paid not on the wages of labour and profits of capital in apple-farming but at the same *ad valorem* rate on the value of the apple-output,  $X_2$ .

<sup>2</sup> If the bees had a bad effect upon the apples, then we should have an external diseconomy, which may be regarded as an unpaid negative factor of production. The bee-keepers, in addition to getting the bee-food free of charge, are also not charged for some damage which they do to the apple-farmers. In what follows  $\epsilon_{x_2}^{x_1}$  would be  $< 0$ , so that  $\epsilon_{l_2}^{x_2} + \epsilon_{c_2}^{x_2} > 1$ .

increase in apple-output and so on in an infinite progression. The final result can be obtained in the following manner. Differentiating the main equations in equations (5), we have

$$\begin{aligned} dx_1 &= \frac{\partial x_1}{\partial l_1} dl_1 + \frac{\partial x_1}{\partial c_1} dc_1 + \frac{\partial x_1}{\partial x_2} dx_2 \\ dx_2 &= \frac{\partial x_2}{\partial l_2} dl_2 + \frac{\partial x_2}{\partial c_2} dc_2 + \frac{\partial x_2}{\partial x_1} dx_1 \end{aligned}$$

If we keep  $c_1$ ,  $l_2$  and  $c_2$  constant ( $dc_1 = dl_2 = dc_2 = 0$ ) but allow  $l_1$  to vary ( $dl_1 \neq 0$ ),  $dx_1$  and  $dx_2$  will give the marginal social net products of  $l_1$  in the two commodities. We obtain

$$\frac{dx_1}{dl_1} = \frac{\frac{\partial x_1}{\partial l_1}}{1 - \frac{\partial x_1}{\partial x_2} \cdot \frac{\partial x_2}{\partial x_1}} \quad \text{and} \quad \frac{dx_2}{dl_1} = \frac{\frac{\partial x_2}{\partial x_1} \cdot \frac{\partial x_1}{\partial l_1}}{1 - \frac{\partial x_1}{\partial x_2} \cdot \frac{\partial x_2}{\partial x_1}}$$

But

$$\begin{aligned} \bar{L}_1 &= l_1 \bar{x}_1 \frac{dx_1}{dl_1} + l_1 \bar{x}_2 \frac{dx_2}{dl_1} \\ &= l_1 \frac{\partial x_1}{\partial l_1} \frac{\bar{x}_1 + \bar{x}_2 \frac{\partial x_2}{\partial x_1}}{1 - \frac{\partial x_1}{\partial x_2} \cdot \frac{\partial x_2}{\partial x_1}} \\ &= L_1 \frac{1 + \frac{X_2}{X_1} \cdot \epsilon_{x_1}^{x_2}}{1 - \epsilon_{x_1}^{x_2} \epsilon_{x_2}^{x_1}} \end{aligned}$$

Similarly, we get the following expressions for the values of the marginal social net products of the other factors.

$$\begin{aligned} \bar{L}_2 &= L_2 \frac{1 + \frac{X_1}{X_2} \epsilon_{x_2}^{x_1}}{1 - \epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2}} \\ \bar{C}_1 &= \epsilon_{c_1}^{x_1} X_1 \frac{1 + \frac{X_2}{X_1} \epsilon_{x_1}^{x_2}}{1 - \epsilon_{x_1}^{x_2} \epsilon_{x_2}^{x_1}} = (C_1 - \epsilon_{x_1}^{x_2} X_1) \frac{1 + \frac{X_2}{X_1} \epsilon_{x_1}^{x_2}}{1 - \epsilon_{x_1}^{x_2} \epsilon_{x_2}^{x_1}} \\ \bar{C}_2 &= (C_2 - \epsilon_{x_2}^{x_1} X_2) \frac{1 + \frac{X_1}{X_2} \epsilon_{x_2}^{x_1}}{1 - \epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2}} \end{aligned}$$

On these expressions we can make the following three comments:

First, remembering that  $L_1 + C_1 = X_1$  and  $L_2 + C_2 = X_2$ , we can see from the above expressions that  $\bar{L}_1 + \bar{L}_2 + \bar{C}_1 + \bar{C}_2 = X_1 + X_2$ . In other words, if the factors were all paid rewards equal to the value of their marginal social net products, this would absorb the whole of the product, neither more nor less. This is due, of course, to the essential constant-returns nature of the production functions at equations (5), from which it can be seen

that if  $l_1, l_2, c_1$  and  $c_2$  were to increase by 10%, then the production conditions would be satisfied if both outputs also increased by 10%. In other words, we are still dealing with a pure unpaid-factor case; there is no adding-up problem for society; every factor can be given a reward equal to the value of its marginal social net product if the revenue from the taxes levied on those which ought to be taxed is used to subsidise the earnings of those which ought to be subsidised.

Secondly,  $\bar{L}_1, \bar{L}_2, \bar{C}_1$  and  $\bar{C}_2$  are all seen to be positive finite quantities, provided that  $\epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2} < 1$ . From the last of equations (5) it can be seen that  $\epsilon_{x_1}^{x_2}$  and  $\epsilon_{x_2}^{x_1}$  are both  $< 1$ ; it requires a 10% increase of land, labour and apple-blossom to increase the output of honey by 10%, so that a 10% increase in the supply of apple-blossom alone will increase the output of honey by less than 10%. But  $\epsilon_{x_2}^{x_1}$  and  $\epsilon_{x_1}^{x_2}$  are both positive, since we are dealing with external economies and not diseconomies. It follows, therefore, that  $0 < \epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2} < 1$ , so that  $\bar{L}_1, \bar{L}_2, \bar{C}_1$  and  $\bar{C}_2$  are all positive finite quantities. It is because  $\epsilon_{x_2}^{x_1}$  and  $\epsilon_{x_1}^{x_2}$  are both positive fractions that the infinite progression of an increase in apple-output causing an increase in honey-output, causing an increase in apple-output and so on, adds up only to a finite sum. For example, if both  $\epsilon_{x_2}^{x_1}$  and  $\epsilon_{x_1}^{x_2}$  are one-half, a 10% increase in apple-output causes a 5% increase in honey-output, but this 5% increase in honey-output causes only a  $2\frac{1}{2}\%$  increase in apple-output; which causes only a  $1\frac{1}{4}\%$  increase in honey-output and so on in a diminishing geometric progression.

Thirdly, from the above expressions for  $L_1$  and  $\bar{L}_1$ , we obtain

$$\frac{\bar{L}_1 - L_1}{L_1} = \frac{\frac{X_2}{X_1} \epsilon_{x_1}^{x_2} + \epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2}}{1 - \epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2}}$$

which shows the *ad valorem* rate of subsidy which must be paid to  $l_1$  to bring its earnings up to the value of its marginal social net product. We can obtain a similar expression for the rates of tax levyable upon  $C_1$ .

$$\frac{C_1 - \bar{C}_1}{C_1} = \frac{\epsilon_{x_2}^{x_1} \frac{X_1}{C_1} - \frac{X_2}{X_1} \epsilon_{x_1}^{x_2} + \frac{X_2 - C_1}{C_1} \epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2}}{1 - \epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2}}$$

Corresponding expressions for  $\frac{\bar{L}_2 - L_2}{L_2}$  and  $\frac{C_2 - \bar{C}_2}{C_2}$  can be obtained by interchanging the subscripts 1 and 2. It can be seen from adding  $C_1 - \bar{C}_1$  and  $C_2 - \bar{C}_2$  that there will be a positive tax revenue raised from capital as a whole. But either  $C_1 - \bar{C}_1$



or  $C_2 - \bar{C}_2$  might be negative, *i.e.*, a subsidy might be payable on the earnings of capital in one of the two industries as well as upon the earnings of labour in both of the industries. For example,  $C_2 - \bar{C}_2$  would be  $< 0$  if  $\epsilon_{x_2}^x$  were very large relatively to  $\epsilon_{x_1}^x$ . This would mean, for example, that the production of honey (industry 1) did very little to help the production of apples (industry 2), while the production of apples did much to help the production of bees. Capitalists in apple-farming should be subsidised because the unpaid benefits which they confer upon the bee-keepers more than outweigh the unpaid benefits which they receive from labour and capital employed in bee-keeping. Indeed, all the results obtained from equations (4) can be obtained from the expressions derived from equations (5) by writing  $\epsilon_{x_2}^x = 0$ .

### V. THE CREATION OF ATMOSPHERE

A distinction must be drawn between a "factor of production" and a physical or social "atmosphere" affecting production. We may take the rainfall in a district as a typical example of atmosphere. The rainfall may be deficient in the sense that a higher rainfall would increase the farmers' output, but nevertheless what rainfall there is will be available to all farms in the district regardless of their number. Thus if in the district in question the amount of land, labour and capital devoted to, say, wheat-farming were to be increased by 10%, the output of wheat would also be increased by 10% even if the rainfall were to remain constant. This is quite different from the case of a factor of production for which no payment is made; in our previous example, a 10% increase in the output of apples (and so in the supply of apple-blossom) would be necessary, in addition to a 10% increase in the amount of land, labour and capital devoted to bee-keeping, if the output of honey is to be increased by 10%. In these examples, rainfall is an "atmosphere" for wheat-farming; but the output of apples is an "unpaid factor of production" for bee-keeping.

The distinction should now be clear. Both a factor of production and an atmosphere are conditions which affect the output of a certain industry. But the atmosphere is a fixed condition of production which remains unchanged for all producers in the industry in question without anyone else doing anything about it, however large or small—within limits—is the scale of operations of the industry. On the other hand, the factor of production is an aid to production which is fixed in amount, and which is therefore available on a smaller scale to each producer in the industry

if the number of producers increases, unless someone does something to increase the total supply of the factor.

The external economies which we have examined in the last section are concerned with factors of production for which the individual producer pays nothing. We must turn now to external economies and diseconomies which are due to the fact that the activities of one group of producers may provide an atmosphere which is favourable or unfavourable to the activities of another group of producers. For example, suppose that afforestation schemes in one locality increase the rainfall in that district and that this is favourable to the production of wheat in that district. In this case the production of timber creates an atmosphere favourable to the production of wheat.

In these cases there is an adding-up problem for society as a whole. There may be constant returns to the factors of production employed in either industry alone. That is to say, a 10% increase in the amounts of land, labour and capital employed in producing wheat might, in any given atmosphere, result in a 10% increase in the output of wheat. And a 10% increase in the amount of land, labour and capital employed in producing timber might, apart from its effect in changing the atmosphere for wheat-farmers, cause a 10% increase in the output of timber. It follows that a 10% increase in the amount of land, labour and capital employed both in the timber industry and in wheat-farming will increase the output of timber by 10% and the output of wheat by more than 10% (because of the improvement in the atmosphere for wheat producers). To society as a whole there are now increasing returns to scale; to pay every factor a reward equal to the value of its marginal social net product will account for more than the total output of the two industries; revenue will have to be raised from outside sources by general taxation if subsidies are to be paid on a scale to bring every factor's reward up to the value of its marginal social net product.

We can express this sort of situation by the following equations :

$$\left. \begin{aligned} x_1 &= H_1(l_1, c_1)A_1(x_2) \\ x_2 &= H_2(l_2, c_2) \end{aligned} \right\} \dots \dots (6)$$

where once more

$$\left. \begin{aligned} \epsilon_{l_1}^{x_1} + \epsilon_{c_1}^{x_1} &= \epsilon_{l_2}^{x_2} + \epsilon_{c_2}^{x_2} = 1. \text{ } ^1 \end{aligned} \right\}$$

<sup>1</sup> Since  $l_1 \frac{\partial H_1}{\partial l_1} = \frac{l_1}{H_1 A_1} \cdot \frac{A_1 \partial H_1}{\partial l_1} H_1 = \frac{l_1}{x_1} \cdot \frac{\partial x_1}{\partial l_1} H_1 = \epsilon_{l_1}^{x_1} H_1$ , we have  $H_1 = \epsilon_{l_1}^{x_1} H_1 + \epsilon_{c_1}^{x_1} H_1$ , so that  $1 = \epsilon_{l_1}^{x_1} + \epsilon_{c_1}^{x_1}$ .

$x_2 = H_2(l_2, c_2)$  is the ordinary competitive constant-returns production function for the timber industry. There is the same type of production function for the wheat industry; but in this case the output due to the use of labour and land ( $H_1[c_1, l_1]$ ) is subject to an atmosphere ( $A_1$ ). If the atmosphere is favourable then  $H_1(l_1, c_1)$  is multiplied up by a large factor to give the actual output ( $x_1$ ). In the case which we are examining the atmosphere for the wheat industry ( $A_1$ ) is made to depend upon the output of the timber industry ( $A_1 = A_1[x_2]$ ).

The atmosphere factor ( $A_1$ ) is thus subject to the following conditions.  $A_1(0) = 1$ , *i.e.*, we define our terms in such a way that  $H_1(l_1, c_1)$  is equal to what the output of wheat would be if there were no timber output.  $A_1$  is always  $> 0$ , *i.e.*, there cannot be so powerful an external diseconomy that the output of the industry affected becomes negative. When  $A_1(x_2) > 1$ , then there is an average external economy, *i.e.*, the output of wheat is greater than it would have been had there been a zero output of timber instead of a positive output ( $x_2$ ); and similarly, when  $A_1(x_2) < 1$ , there is an average external diseconomy. When  $A_1'(x_2)$  is  $> 0$ , then there is a marginal external economy, *i.e.*, the output of wheat would be improved by a further increase in the output of timber; and when  $A_1'(x_2)$  is  $< 0$ , there is a marginal external diseconomy.

The actual rewards of the factors of production are easily seen to be  $L_1 = \epsilon_{l_1}^{x_1} X_1$ ,  $C_1 = X_1 - L_1 = \epsilon_{c_1}^{x_1} X_1$ ,  $L_2 = \epsilon_{l_2}^{x_2} X_2$  and  $C_2 = X_2 - L_2 = \epsilon_{c_2}^{x_2} X_2$ . In the case of the factors employed in wheat-farming (industry 1) there will be no divergence between the reward paid and the value of the marginal social net product; and  $L_1 = \bar{L}_1$  and  $C_1 = \bar{C}_1$ .

But the rewards actually paid to the factors of production in the timber industry (industry 2) will be lower than the value of their marginal social net products because they will not be paid for the favourable atmosphere which they create for wheat farmers. Thus

$$\begin{aligned} \bar{L}_2 &= \epsilon_{l_2}^{x_2} X_2 + l_2 \bar{x}_1 \frac{\partial x_1}{\partial x_2} \cdot \frac{\partial x_2}{\partial l_2} \\ &= L_2 \left( 1 + \frac{X_1}{X_2} \cdot \epsilon_{x_2}^{x_1} \right) \end{aligned}$$

where  $\epsilon_{x_2}^{x_1} = \frac{x_2}{x_1} \cdot \frac{\partial x_1}{\partial x_2}$  or  $\frac{x_2}{A_1} \cdot \frac{\partial A_1}{\partial x_2}$ , the percentage increase in the output of wheat which would be brought about by a 1% increase in the output of timber through the improvement in the atmos-

phere for wheat production. And similarly, it can be shown that  $\bar{C}_2 = C \left(1 + \frac{X_1}{X_2} \epsilon_{x_2}^{x_1}\right)$ . In other words, the earnings of both  $l_2$  and  $c_2$  or, alternatively, the price of the product  $x_2$  must be subsidised from general revenue at the *ad valorem* rate of  $\frac{X_1}{X_2} \epsilon_{x_2}^{x_1}$  if all factors are to receive rewards equal to the value of their marginal social net products.

As in the case of unpaid factors, these reactions of one industry upon the other may be reciprocal. Industry 1 may create a favourable or unfavourable atmosphere for industry 2, as well as industry 2 for industry 1. In this case we have

$$\left. \begin{aligned} x_1 &= H_1(l_1, c_1)A_1(x_2) \\ x_2 &= H_2(l_2, c_2)A_2(x_1) \end{aligned} \right\} \dots \dots (7)$$

where

$$\epsilon_{l_1}^{x_1} + \epsilon_{c_1}^{x_1} = \epsilon_{l_2}^{x_2} + \epsilon_{c_2}^{x_2} = 1$$

Here again  $L_1 = \epsilon_{l_1}^{x_1} X_1$ ,  $C_1 = X_1 - L_1 = \epsilon_{c_1}^{x_1} X_1$ ,  $L_2 = \epsilon_{l_2}^{x_2} X_2$  and  $C_2 = X_2 - L_2 = \epsilon_{c_2}^{x_2} X_2$ .

But when we come to consider the marginal social net product, we have to take into account the infinite chain of action and reaction of the one industry upon the other, as in the case of apple-growing and bee-keeping examined above. The marginal social net product of  $l_1$ , for example, is obtained by differentiating the first two of equations (7), keeping  $l_2$ ,  $c_1$  and  $c_2$  all constant. We obtain

$$\frac{dx_1}{dl_1} = \frac{\epsilon_{l_1}^{x_1} \cdot \frac{x_1}{l_1}}{1 - \epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2}} \quad \text{and} \quad \frac{dx_2}{dl_1} = \frac{\epsilon_{l_1}^{x_1} \epsilon_{x_1}^{x_2} \cdot \frac{x_2}{l_1}}{1 - \epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2}}$$

Now  $\bar{L}_1 = l_1 \bar{x}_1 \frac{dx_1}{dl_1} + l_1 \bar{x}_2 \frac{dx_2}{dl_1}$ , so that  $\frac{\bar{L}_1}{L_1} = \frac{1 + \frac{X_2}{X_1} \epsilon_{x_1}^{x_2}}{1 - \epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2}}$

Similarly, we can show that  $\frac{\bar{C}_1}{C_1} = \frac{\bar{L}_1}{L_1}$  and that

$$\frac{\bar{L}_2}{L_2} = \frac{\bar{C}_2}{C_2} = \frac{1 + \frac{X_1}{X_2} \epsilon_{x_2}^{x_1}}{1 - \epsilon_{x_2}^{x_1} \epsilon_{x_1}^{x_2}}$$

In other words, in order that each factor should obtain a reward equal to the value of its marginal social net product both labour and capital in industry 1, or alternatively, the price of the product

of industry 1, should be subsidised at the *ad valorem* rate of

$$\frac{X_2}{X_1} + \epsilon_{x_1}^{x_2};$$

and similarly, in industry 2 a rate of subsidy

$$\frac{X_1}{X_2} + \epsilon_{x_2}^{x_1}$$

of  $\frac{X_1}{X_2} + \epsilon_{x_2}^{x_1}$  should be paid.

So far throughout this note we have assumed that in all external economies or diseconomies, whether of the unpaid-factor or of the atmosphere-creating kind, it is the *output* of one industry which affects production in the other. But this is, of course, not necessarily the case. It may be the employment of one *factor* in one industry which confers an indirect benefit or the reverse upon producers in the other industry.<sup>1</sup> Moreover, in the case in which atmosphere is created, the output of industry 2 may create an atmosphere for industry 1 which increases the efficiency of a particular *factor* in industry 1 rather than the general level of *output*.<sup>2</sup> Or the employment of a particular *factor* in industry 2 might create conditions which improved the efficiency of a particular factor in industry 1.<sup>3</sup> And any combination of these indirect effects of industry 2 upon industry 1 might be combined with any other combination of such effects of industry 1 on industry 2. Clearly, we cannot consider in detail all the very many possibilities.

But consider the following particular case :

$$\left. \begin{aligned} x_1 &= H_1(\lambda_1, c_1) \\ x_2 &= H_2(\lambda_2, c_2) \\ \lambda_1 &= l_1 A(l) \\ \lambda_2 &= l_2 A(l) \\ l &= l_1 + l_2 \end{aligned} \right\} \dots \dots \dots (8)$$

where  $l_1$  = the number of workers employed in industry 1 and  $\lambda_1$  = the equivalent number of workers of an efficiency which an individual worker would have if the total labour force were very small ( $l \rightarrow 0$ , so that  $A \rightarrow 1$ .)

This is the case where the total labour force in the two industries ( $l$ ) affects the general efficiency of labour. We may suppose that, up to a certain point, a growth in the absolute size

<sup>1</sup> In this case we should have equations of the type of  $x_1 = H_1(l_1, c_1, l_2)$  in the case of unpaid factors, and of the type of  $x_1 = H_1(l_1, c_1)A_1(l_2)$  in the case of atmosphere-creation.

<sup>2</sup> In this case the equations would be of the type  $x_1 = H_1\{l_1 A_1(x_2), c_1\}$ .

<sup>3</sup> For example,  $x_1 = H_1\{l_1 A(c_2), c_1\}$ .

of the labour force employed in these two industries causes a general atmosphere favourable to the efficiency of labour by enabling workers to communicate to each other a certain know-how about, and interest in, the mechanical processes which are common to the two industries.

Now the individual employer in any one firm in either industry will regard A as being unaffected by his own actions, because the indirect effect which an increase in the number of workers employed by him alone will have upon the general efficiency of his own labour will be a negligible quantity. He will go on taking on labour of any given level of efficiency until the wage paid to a unit of labour is equal to the price paid for its marginal product at that level of efficiency. In other words,

$$L_1 = \lambda_1 \bar{x}_1 \frac{\partial x_1}{\partial \lambda_1} = \epsilon_{\lambda_1}^{x_1} X_1$$

The reward paid to  $c_1$  will be  $C_1 = X_1 - L_1 = \epsilon_{c_1}^{x_1} X_1$ .

Similarly,  $L_2 = \epsilon_{\lambda_2}^{x_2} X_2$  and  $C_2 = \epsilon_{c_2}^{x_2} X_2$ .

In this case  $\bar{C}_1 = \epsilon_{c_1}^{x_1} X_1$  and  $\bar{C}_2 = \epsilon_{c_2}^{x_2} X_2$  because there are no external economies or diseconomies involved in decisions to apply more capital in either industry, so that  $C_1 = \bar{C}_1$  and  $C_2 = \bar{C}_2$ .

But in evaluating the value of the marginal social net product of labour we have to take into account the effect which the employment of more labour by one particular employer may have upon the efficiency of labour for all other employers in industry 1 and for all other employers in industry 2. The value of the marginal social net product exceeds the wage which will be offered for it by these two sums, so that

$$\bar{L}_1 = L_1 + l_1 \bar{x}_1 \frac{\partial x_1}{\partial \lambda_1} \cdot \frac{\partial \lambda_1}{\partial l} \cdot \frac{\partial l}{\partial l_1} + l_1 \bar{x}_2 \frac{\partial x_2}{\partial \lambda_2} \cdot \frac{\partial \lambda_2}{\partial l} \cdot \frac{\partial l}{\partial l_1}$$

Since  $\frac{\partial l}{\partial l_1} = 1$ ,  $\frac{\partial \lambda_1}{\partial l} = \frac{\lambda_1}{A} \cdot \frac{\partial A}{\partial l}$ , and  $\frac{\partial \lambda_2}{\partial l} = \frac{\lambda_2}{A} \cdot \frac{\partial A}{\partial l}$ ,

we have  $\bar{L}_1 = L_1 + (L_1 + L_2) \frac{l_1}{l} \epsilon_l^A$ , where  $\epsilon_l^A = \frac{l}{A} \cdot \frac{dA}{dl}$ .

Similarly,  $\bar{L}_2 = L_2 + (L_1 + L_2) \frac{l_2}{l} \epsilon_l^A$ . Now if the wage-rate is the

same in both industries so that  $\frac{l_1}{l} = \frac{L_1}{L_1 + L_2}$  and  $\frac{l_2}{l} = \frac{L_2}{L_1 + L_2}$ ,

we have  $\frac{\bar{L}_1}{L_1} = \frac{\bar{L}_2}{L_2} = 1 + \epsilon_l^A$ . The employment of labour in both industries must be subsidised at the *ad valorem* rate of  $\epsilon_l^A$ , if rewards are to be raised to the value of marginal social net products.

## VI. CONCLUSION

It is not claimed that this division of external economies and diseconomies into unpaid factors and the creation of atmosphere is logically complete. External economies exist whenever we have production functions of the form

$$\begin{aligned}x_1 &= F_1(l_1, c_1, l_2, c_2, x_2) \\x_2 &= F_2(l_2, c_2, l_1, c_1, x_1)\end{aligned}$$

where  $F_1$  and  $F_2$  are not necessarily homogeneous of the first degree. But it is claimed that it may clarify thought on different types of external economy and diseconomy to distinguish thus between: (1) those cases in which there are constant returns for society, but not necessarily constant returns in each industry to the factors which each industry employs and pays for, and (2) those cases in which there are constant returns in each industry to those factors which it controls and pays for, but in which there are not constant returns for the two industries taken together, the scale of operations being important in the one industry because of the atmosphere which it creates for the other. One of the most important conclusions to be drawn is that in the case of type (1)—the unpaid-factor case—there is no adding-up problem for society as a whole; in order to pay every factor a reward equal to the value of its marginal social net product some factors must be taxed and others subsidised, and the revenue from the appropriate taxes will just finance the expenditure upon the appropriate subsidies. But in the case of the creation of atmosphere (type (2)) the subsidies (or taxes) required to promote (or discourage) the creation of favourable (or unfavourable) atmosphere are net additions to (or subtractions from) society's general fiscal burden. But, in fact, of course, external economies or diseconomies may not fall into either of these precise divisions and may contain features of both of them.

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