Confuser Surplus

By EDWARD R. MOREY*

Paul Samuelson noted that consumer's surplus is a topic "...about which an earlier generation of economists were able to indulge in much argumentation" (1947, p. 195). Indulgence persists. The current consumer's surplus literature shows that there is still much for authors to argue about and to confuse readers.[1] This paper begins and ends with what I consider to be the fundamental question for users of the consumer's surplus concept. Do dollar measures of utility changes exist? The answer is seen to hinge critically on what one means by "measures," and on whether one assumes an ordinal or a cardinal preference ordering.

I. Confusing and Seemingly Contradictory Statements: Why?

The evidence on the question, in terms of quotes from the great and almost great appears mixed.[2] In defense of dollar measures of utility changes, Alfred Marshall states

> ...[The consumer] derives from a purchase a surplus of satisfaction. The excess of the price which he would be willing to pay rather than go without the thing, over that which he actually does pay is the economic [money] measure of this surplus of satisfaction. It may be called consumer's surplus.
> [1920, p. 124]

In agreement, John Hicks states "...the best way of looking at consumer's surplus is to regard it as a means of expressing, in terms of money income, the gain which accrues to the consumer as a result of a price fall" (1943, p. 40). Alternatively, Samuelson has a section in *Foundations* entitled, "Why Consumer's Surplus Is Superfluous," in which he finds the concept worthless (1947, pp. 195–97).

More recently, Eugene Silberberg; Donald Richter; Michael Burns; Arnold Harberger; Robert Willig; G. W. McKenzie and I. F. Pearce; and Charles Cicchetti et al. to name a few, have added to the debate. Silberberg states "The simple truth is that there is no unique dollar or money equivalent of a change in utility" (1978, p. 353). In some sort of agreement, Richter notes that "...it is fruitless to try to justify a consumer welfare measure on the grounds that it represents a money index of utility change..." (1977, p. 502). Burns states "we recognize the impossibility of associating a unique evaluation in money terms with the utility difference between any two situations..." (1973, p. 341). Harberger states "That this [i.e., the notion that consumer's surplus should measure changes in utility directly] would be a fruitless pursuit should be obvious..." (1971, p. 788, fn. 3), and most recently, McKenzie and Pearce state "attempts to measure...consumer's surplus...are no more than disguised attempts to measure utility itself. Each can be correctly measured only by measuring utility" (1982, p. 673).

There are, however, other recent statements in the works of these authors and others which suggest that consumer's surplus measures are dollar measures of utility changes. Burns, for example, makes other statements that suggest that the consumer's surplus concept is a useful dollar measure of utility changes. At the beginning of Willig's often-cited article he refers to the compensating and equivalent variation (both measured

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[1] To limit the scope of confusion, I consider only the measurement of utility changes for a utility-maximizing individual in a single time period.

[2] I find a lot of the statements about consumer's surplus seemingly contradictory. Whether they are when interpreted correctly is another matter. The intent of this section is not to suggest that the quoted authors are confused, rather my intent is to show how easily the nonspecialist, like myself, can become confused.

in dollars) as "...the correct theoretical measures of the welfare impact of changes in prices and income on an individual" (1976, p. 589). He concludes the paper by stating that "...at the level of the individual consumer, cost-benefit welfare analysis can be performed rigorously and unapologetically by means of consumer's surplus" (p. 596). Harberger explicitly suggests that consumer's surplus is a dollar measure of utility change. He states

> ...[Equation] (5) in effect converts the change in utility into monetary terms by dividing it by the marginal utility of income. There is obviously no problem when the latter is not changing, but when it does change as a consequence of the action(s) being analyzed, the conversion of utility into money is implicitly carried out at the midpoint of the beginning and ending marginal utilities of income. The criticism that consumer-surplus concepts have validity only when the marginal utility income is constant must be rejected.
> [pp. 788–89]

The work of Cicchetti et al. also definitely suggests that dollar measures of benefits (utility) exist. In a nice mix of the theoretical and the applied, they derive a consumer's surplus measure and use it to measure the dollar "benefits" associated with a new ski area. In apparent agreement, McKenzie and Pearce state that "...the equivalent variation ...is an index of utility" (1982, p. 673). It is clear that the "stated" evidence is mixed regarding whether dollar measures of utility changes exist.

Why has this confusion persisted for so long? These seemingly contradictory statements coexist for four reasons:

1) People are often not explicit about whether they believe consumers have *cardinal* or *ordinal* preference orderings.

2) The term *measures* in the expression *dollar measures* of utility changes has not been clearly defined.

3) The usefulness of consumer's surplus measures depends in part on whether one specifies systems of demand equations directly, or whether one begins by specifying the utility function and then deriving the corresponding system of demand equations. Consumer's surplus measures are more useful to advocates of the first approach than they are to advocates of the latter approach, but both groups tend to forget this.

4) The usefulness of the cardinal properties of consumer's surplus measures depends in part on whether one assumes that all aspects of the alternatives were considered or whether one takes the other view that many important components (characteristics, political implications, etc.) are omitted in the calculation of consumer's surplus measures. The cardinal properties are more important to those who hold the latter view.

The following section discusses preferences and measurement while Section III discusses what properties money indices of utility should have and what properties they do have. Section IV takes up the third reason for the confusion, namely that there are different approaches to demand estimation. Section V considers the calculation of money measures. The fourth reason for confusion is discussed in Section VI. The paper closes in Section VII with an example.

II. Preferences and Measurement

This section defines ordinal preferences and cardinal preferences and discusses the measurement of preference with numbers.[3]

Consider the bundle of goods X_i as one element in the set of all possible bundles. Bundles, like all elements in all sets, possess characteristics. One of the characteristics possessed by all bundles, at least from the individual's point of view, is preference (Pareto's "ophelimity"). What we mean by preference depends on whether we attach the adjective ordinal, weakly cardinal or strongly cardinal.

A. *Ordinal Preferences*

Ordinal preference has magnitude; that is, preference for one bundle is capable of being more than, less than, or equal in magnitude to the preference for some other bundle. If given the choice between two bundles (X_i

[3]The theory and logic of measurement, like consumer's surplus, is only simplistic when first encountered. See W. S. Torgerson (1958, pp. 1–40) and Bertrand Russell (1938, pp. 157–87).

and X_j), if X_i is chosen ($X_i P X_j$), then the magnitude of preference associated with X_i is defined to be greater than the magnitude of preference associated with bundle X_j. If the individual is indifferent between X_i and X_j ($X_i I X_j$), then the bundles are defined to have equal magnitudes of preference. If an individual is capable of ordering the bundles on the basis of preference magnitudes, and if that ordering is complete, reflexive, and transitive, then that individual has at least an ordinal preference ordering.

One can measure preferences that are solely ordinal on a numerical scale. To measure means to associate with each magnitude of preference a symbol such that the relationships between the symbols reflect accurately the corresponding relationships between the different magnitudes of preference. Ordinal magnitudes of preference only order the bundles so the symbols need only reflect that ordering; that is, if $X_i P X_j$, then the symbol associated with X_i must take precedence over the symbol associated with X_j. Though not conventional, Ernest Phelps Brown (1934) showed that words could be used to measure magnitudes of ordinal preference; for example, if $X_i P X_j$, this could be reflected by associating the word "duck" with X_i and the word "economist" with X_j since duck takes precedence over economist (it comes first in the dictionary).

Numbers will also work but one must be careful. One can define a utility function $U = \phi(X)$ that associates a number with each bundle such that if $X_i P X_j$, then $\phi(X_i) > \phi(X_j)$, and if $X_i I X_j$, then $\phi(X_i) = \phi(X_j)$. All numerical scales have certain properties, but in this case most of these properties are meaningless. The danger is that this is easy to forget. Numbers have names, numbers are ordered, the difference between two numbers is a number (so differences between numbers are ordered), the ratio of two numbers is a number, and the series of numbers has a unique origin indicated by the number "zero." Every numerical scale will have all of these properties regardless of whether they are appropriate for the underlying characteristics being measured. To measure ordinal preferences, one only needs the ordering property of numbers. For this reason, a numerical measurement of ordinal preferences ($\phi(X)$) is unique only up to a monotonic transformation. The function $F(\phi)$, where $\partial F(\phi)/\partial\phi > 0$, defines the class of monotonic transformations of $\phi(X)$.

B. *Weakly Cardinal Preferences*

Assume that, besides being able to rank bundles, the individual is able to ordinally rank the preference differences between bundles. That is, the individual knows whether a move from X_i to X_j is preferred, not preferred, or equally preferred to a move from X_k to X_l. Call this the ability to rank differences. If the individual has this ability, then the difference between two magnitudes of preference is also a magnitude of preference, implying preference has intensity. This is not the case if preferences are solely ordinal. If the ordering of differences is complete, reflexive, and transitive, then preferences are said to be weakly cardinal (Nicholas Georgescu-Roegen, 1968, p. 260).

If one is willing to stipulate some additional axioms, then weakly cardinal preferences can be measured numerically.[4] That is, a function $\phi(X)$ exists such that: 1) $\phi(X_i) \gtreqless \phi(X_j)$ if, respectively, $X_i P X_j$, $X_i I X_j$, or $X_j P X_i$; and 2) $[\phi(X_j)-\phi(X_i)] \gtreqless [\phi(X_l)-\phi(X_k)]$ if, respectively, $(X_i \text{ to } X_j) P (X_k \text{ to } X_l)$, $(X_i \text{ to } X_j) I (X_k \text{ to } X_l)$, or $(X_k \text{ to } X_l) P (X_i \text{ to } X_j)$.[5] To numerically measure weakly cardinal preferences, we need only the ordering property of numbers and the fact that the difference between two numbers is also a number thus is also ordered. For this reason, a numerical measure of weakly cardinal preference ($\phi(X)$) is unique only up to an affine transformation ($\alpha + \beta\phi(X)$ defines the class of affine transformations of $\phi(X)$). At this

[4]Samuelson (1938) showed that a function $\phi(X)$ that ranks bundles and differences correctly will not exist (Oscar Lange, 1934a, b, thought otherwise) unless some restrictions are imposed on the individual's ordering of differences. One necessary restriction is that if $(X_i \text{ to } X_j) I (X_l \text{ to } X_m)$ and if $(X_j \text{ to } X_k) I (X_m \text{ to } X_n)$, then $(X_i \text{ to } X_k) I (X_l \text{ to } X_n)$. The mathematical function $\phi(X)$ will order differences as if this restriction on preferences is appropriate. If it is not appropriate, the function's ranking of differences will be incorrect.

[5]As Phelps Brown notes, one can also measure weakly cardinal preferences with words (for example, English words to order bundles and French words to order differences).

point one must be careful; the ratio $[\phi(X_j)-\phi(X_i)]/[\phi(X_l)-\phi(X_k)]=r$ is well-defined numerically (it might, for example, be two) but that does not mean that the move from (X_i to X_j) is r times preferred to the move from (X_k to X_l). The ratio of two magnitudes of preference is not defined if preferences are only weakly cardinal. One cannot, therefore, attribute meaning to the ratio properties of the numerical scale.[6]

C. *Strongly Cardinal Preferences*

One must assume that preferences are strongly cardinal before one can meaningfully say that the move from (X_i to X_j) is preferred twice as much as the move from (X_l to X_k). Preferences are strongly cardinal if, in addition to being weakly cardinal, the ratio of two magnitudes of preference differences is also a meaningful magnitude of preference.[7] As with weakly cardinal preferences, the utility function ($\phi(X)$) is unique only up to an affine transformation if preferences are strongly cardinal, but now the ratio $[\phi(X_j)-\phi(X_i)]/[\phi(X_l)-\phi(X_k)]=r$ does reflect a meaningful property of the individual's preferences.

III. What Properties Should a Money Index of Utility Have? What Properties Do they Have?

The answer to the first question depends on whether one believes the consumer has a cardinal preference ordering (weak or strong) or just an ordinal ranking of bundles. The cardinalist would like the money index of utility to be an affine transformation of the utility function. Hence, if the move from (X_i to X_j) provides A utils, the move from (X_l to X_m) provides B utils, and the dollar measure of the change from (X_i to X_j) is \$10, the cardinalist would like the dollar measure of the change from (X_l to X_m) to be (B/A)\$10. That is, the cardinalist would like the dollar index of utility changes to have the same properties as utility.

Alternatively, the ordinalist only requires that the money index rank alternative bundles in the same order the consumer would. The ordinalist only requires that the money index be a strictly monotonic transformation of the utility function. The cardinal magnitude of the difference in utility levels associated with different alternatives can tell one nothing about an ordinalist's intensity of preferences because the ordinalist's preferences do not have intensities.

Concerning what properties money indices do have, is there a money index of utility that is a strictly monotonic transformation of the utility index? The answer is yes. The equivalent variation (EV)[8] will rank bundles in the same order the consumer would. In this sense, dollar measures of utility changes exist. McKenzie and Pearce express this view when they state that "the equivalent variation...is an index of utility" (1982, p. 673). This is what Willig means when he tells us that consumer's surplus is a useful concept.

Is there, however, a money index of utility that is an affine transformation of a cardinally meaningful utility index? That is, does the function $M=\alpha+\beta U$, where α and β are constant scalars, M is money, and U is the cardinal utility index, exist? The answer, in

[6]Historically, Lange (1934a,b) thought that if the individual can rank bundles and differences then the individual can automatically associate meaning to the ratios of preference magnitudes and he or she can say one move is r times more preferred than some other move. Phelps Brown showed this assertion to be incorrect. This finding is reiterated by Georgescu-Roegen.

[7]A stronger condition comes to mind. One could additionally assume that the ratio of preference magnitudes associated with two bundles (as opposed to the ratio of the preference differences between bundles) is also a meaningful magnitude of preference. One could then say that bundle X_i provides n times as much preference as bundle X_j. Call preferences with this property purely cardinal. In this case, the utility function is only unique up to a linear transformation ($\beta\phi(X)$ defines the class of linear transformations of $\phi(X)$). Purely cardinal preferences are not common in the literature.

[8]The equivalent variation associated with a change from the parametric price-income configuration (P^0, M^0) to the parametric price-income configuration (P', M') is $EV=E[U',P^0]-E[U',P']+M'-M^0$ where $E[\cdot]$ is the expenditure function and U' is the maximum utility level given P' and M'. The value of the EV is invariant to monotonic transformations of the expenditure function. If the indirect utility function has been identified up to a monotonic transformation, then the expenditure function can easily be obtained by solving the indirect function for the level of expenditures.

general, is no, as is easy to see.[9] Such a function will exist only if β is a constant scalar that transforms utils into money; β is therefore the inverse of the marginal utility of money and will be a constant only if the marginal utility of money ($\partial U/\partial M$) is constant. The marginal utility of money is not usually assumed to be constant.[10] In this sense, dollar measures of utility changes do not exist. This is what I believe Silberberg, Richter, and Burns meant in the above quotes.

It is now clear that a money index of utility (i.e., the EV) exists which is a strictly monotonic transformation of utility, but remember that the cardinal magnitudes of the EVs cannot, in general, tell us anything about an individual's intensity of preference. The fact that the EVs are invariant to monotonic transformations of the utility function proves this. Changing a cardinal utility function by a monotonic transformation will change the intensity of preferences, but not the EVs.

IV. The Usefulness of Money Measures Depends in Part on How You Estimate Demand Equations

There are two distinct approaches. The first begins by specifying an explicit algebraic form for either the direct or indirect utility function and then derives the corresponding system of demand equations; the second specifies the algebraic form of the demand equations directly. If one adopts the first approach, estimation of the demand equations implies direct knowledge of an estimated utility function. One can then easily use this estimated utility function to ordinally rank projects. Advocates of this approach do not need money measures of utility changes to rank projects and sometimes forget that advocates of the other approach might. This forgetfulness is one of the reasons Samuelson used the adjective "superfluous" (1947, p. 195).

However, if one adopts the second approach, knowledge of the demand functions does not immediately imply knowledge of the ordinal preference ordering. One must solve the integrability problem to obtain the preference ordering. This can be difficult. Advocates of this direct approach to demand estimation are therefore attracted to money measures for their ability to rank projects without direct knowledge of the preference ordering. This group sometimes forgets that the other group does not need these money measures to rank projects.

This forgetfulness probably motivated the following responses to Samuelson's remarks about superfluousness. I. M. D. Little states

> This is a rather surprising passage [referring to Samuelson's remarks]. It implies not only that all individuals are "economic", but also that we have a copy of everyone's preference field filed away.... We can readily agree that consumer's surplus is surplus in formal logic, but the point of the theory surely was to establish a practical criteria. [1950, p. 176]

More recently, in the same view, J. M. Currie et al. state

> Samuelson chose to attack the concept [of consumer's surplus] at a purely theoretical level on the assumption that everyone's preference field is known, while the main appeal of the concept ...lies in its relationship with the demand curve. Given that the economist does not have a copy of everyone's preference map filed away, it does not really diminish the practical significance of the concept to argue.... [1971, pp. 786–87]

[9]For a less intuitive proof, see Silberberg's (1972) line integral proof.

[10]Samuelson (1942) has proven that the marginal utility of money cannot be a constant independent of both the income and price levels. However, it can be independent of either the income level or some of the prices. The special case where the marginal utility of money ($\partial U(y,p)/\partial y$) is independent of the income level y is often discussed. The marginal utility of money is independent of y iff $U=V(y,p)=y/e(p)$. Note that $V(y,p)=y/e(p)\Leftrightarrow E(U,p)=Ue(p)\Leftrightarrow U(X)$ is homogeneous of degree one in X. If the price level is constant, M and U will be affine transformations of one another iff $V(y,p)=y/e(p)$. This means that the EVs for a group of alternatives will measure the intensity of preferences for those alternatives if the alternatives differ only in terms of the income level and if the individual's cardinal utility function is homogeneous of degree one in X.

As Abram Bergson (1975, p. 38) points out, knowledge of the demand equations (if they fulfill the regularity conditions) is sufficient information, at least in theory, to determine the ordinal preference ordering. One wonders whether Little and Currie et al. do not realize this or whether they just feel that, given the demand equations, it is easier to rank alternatives using consumer's surplus measures than it is to solve the integrability problem directly.

With both approaches to demand estimation, one can only be sure that the ranking of alternatives is correct if one is sure they have deduced the correct demand functions from the market data. On this point, one can never be sure, not even in the world where demand has no stochastic component.

The one approach specifies an algebraic form for the utility function, derives the corresponding system of demand equations, and only then uses data to estimate the coefficients in those demand equations. Imposing an algebraic form on the utility function with complete ignorance of the data amounts unfortunately to imposing a priori restrictions on the preference ordering. These a priori restrictions are embedded in the demand equations. Since their imposition was not based on the data, the estimated demand equations are not necessarily correct and the resulting ranking could be wrong. This is why advocates of this approach often attempt to choose an algebraic form that imposes as few a priori restrictions on the preference ordering as possible while still keeping the form simple enough to econometrically estimate the parameters in the corresponding demand equations. This explains the current popularity of "flexible functional forms" over historically popular forms such as the Bergson and the Linear Expenditure System. Most flexible functional forms (for example, the Translog and Generalized Leontief) are second-order approximations to the true function at some point of approximation. The derived demand equations are therefore first-order approximations. Unfortunately, not even flexible forms are perfectly flexible over the whole range of the data, so restrictions are inevitably imposed on the preference ordering. There is no free lunch when it comes to algebraic forms.

The other approach to demand estimation specifies the algebraic form for the demand equations directly and then uses the market data to estimate their coefficients. While not as obvious, this approach also imposes, perhaps incorrectly, a priori restrictions on the preference ordering. The algebraic form of the demand equations determines the form of the utility function up to a monotonic transformation. Advocates of this approach should attempt to choose, subject to the regularity and simplicity constraints, the most flexible algebraic forms for the demand equations. Again the best we can hope for is algebraic forms that will approximate the true system of demand equations.

Both approaches suffer from the deficiency that they impose a priori restrictions on the preference orderings. This deficiency cannot be avoided, or said differently, we cannot estimate a perfectly flexible system of demand equations with market data. Even in a world where demand has no stochastic component, an estimated demand system (i.e., one that fits all the observations exactly) will not be unique if the number of market observations is finite, as there are an infinite number of functions that pass through a finite number of points. There is no way of knowing that you have the correct one. One cannot even know that one has the correct function in the neighborhood of some chosen point.

V. Using Money Measures to Rank Alternatives without Direct Knowledge of the Preference Ordering

Suppose you have estimated an individual's system of demand equations and are willing to assume it is correct. How can this information be used to determine how the individual would rank a number of price-income configurations? Attempting to solve the integrability problem is not the easiest way to proceed.

We know that *EV*s will rank the alternatives correctly.[11] As noted earlier, *EV*s are

[11] Both the compensating variation (CV) and equivalent variation (EV) will rank bundles correctly. However, the CVs associated with the moves from X_i to X_j and from X_i to X_k will not necessarily rank the two moves correctly, whereas the EVs will.

easy to determine if the expenditure function or indirect utility function is known up to a monotonic transformation, but this information is now unavailable.

McKenzie and Pearce (1976, 1982) have developed a technique to determine the EVs for different alternatives using only the demand functions. Stated simply, they use a Taylor-series expansion to determine the EV for a given alternative as a function of the initial price and income level (y^0, p^0), the changes in prices (Δp), and the change in money income (Δy), where all the coefficients in the Taylor-series expansion are first- and higher-order partial derivatives of the demand equations evaluated at (y^0, p^0).

The other major technique for determining an EV, when the expenditure function has not been identified, is to approximate it with the Marshallian consumer's surplus measure. Willig argues that "in most applications the error of the approximation will be very small" (p. 589). The McKenzie-Pearce technique does not involve approximation errors if the Taylor-series expansion is evaluated in full. Also note that the McKenzie-Pearce technique does not require complete knowledge of the demand equations, it only requires knowledge of the demand equations in the neighborhood of (y^0, p^0). This gives it a definite informational advantage over the Marshallian measure, which requires knowledge of the demand equations over the complete range of the price variations. The McKenzie-Pearce technique is new, but it should prove quite useful to applied welfare economists.

It has been noted that the cardinal properties of an individual's cardinal preference ordering cannot be inferred from the cardinal properties of a money measure of that ordering. The reader, however, should be aware that this statement can be strengthened. One can never determine the cardinal properties of the individual's cardinal preference ordering (i.e., the individual's intensity of preferences) solely on the basis of the individual's behavior in the market.[12] It cannot be done because any monotonic transformation of the true cardinal utility function will explain behavior just as well as the true function.[13] The true function can only be identified if the individual provides, under interrogation, what Samuelson refers to as "introspective information" (1974, p. 1258) on the intensity of their preferences.

However, since individuals possess introspective information about themselves, they can, after being informed of their own EVs for a number of alternatives, use this information to determine their own intensity of preferences for those different alternatives.[14] One uses these dollar amounts in the same way temperature readings can be used to determine how much Palm Springs is preferred at 90°F to Palm Springs at 112°F.

VI. But Don't the Cardinal Properties of the *EV*s Mean Something?

This is a difficult question so before addressing it, let me summarize some pertinent points. 1) Alternatives can be ranked correctly on the basis of their respective EVs (a money measure) if all aspects of the alternatives are considered in the determination of those EVs. 2) Money measures of changes in utility, such as the EV, have cardinal properties independent of whether the underlying preference ordering is cardinal or ordinal. 3) Knowledge of the cardinal differences be-

[12]A point known to Pareto; elucidated by Lange (1934a, pp. 224–25), and Roy G. D. Allen (1935, p. 155). This was brought to my attention by the referee.

[13]This makes one wonder. If cardinal preferences don't seem to help us to choose, why have them (if in fact we do have them)? A major thrust of utility theory since Antonelli in 1886, Fisher in 1892, and Pareto, has been to point out that one doesn't need cardinality to explain market behavior. There has been resistance. For a while some welfare economists thought cardinal utility was required to make interpersonal welfare judgements. Bergstrom (1938) showed that they were wrong. More recently, von Neumann and Morgenstern (1964) and others have argued that certain behavior (i.e., gambling) might imply that individuals have cardinal utility. However, the general trend has been to purge the concept from all but principles books. Why then do I imagine, or hope, that my preference ordering is at least weakly cardinal over some range? Maybe one is just better off with cardinal preferences.

[14]One might wonder why an individual would find it useful to have someone else tell them what their own EVs are. If one knows their own preferences and has complete information concerning all the alternatives, there is no reason. However, if an individual does not have complete information, he will find EVs calculated using complete information useful summary measures.

tween *EV*s is not necessary to rank projects. 4) The cardinal magnitudes of *EV*s are invariant to monotonic transformations of the utility function. 5) Knowledge of another person's *EV*s for a number of alternatives tells us nothing about that individual's intensity of preferences for those alternatives. 6) The *EV*s express how much better off each alternative will make us in dollars evaluated at current prices, and these dollar amounts can be compared cardinally in the sense that we can say $100 is twice as much money as $50. 7) If one has cardinal preferences, one can use one's *EV*s for a number of alternatives to determine one's own intensity of preferences for those alternatives. 8) However, you do not need to be informed of your own *EV*s if you know your own preferences and have complete information on all the alternatives.

With these points in mind, the question boils down to how important it is to know that an *EV* of $100 is twice as much as an *EV* of $50. Not everyone agrees on the answer; some take a theoretical perspective while others adopt a more practical point of view.

Stated simply, in the theoretical view the cardinal properties of the *EV* measures have meaning (in the sense of points 6 and 7), but the cardinal information provided is of little use to policymakers.[15] In a world of ordinal preference ordering, all the decision maker needs is a ranking. In a world of cardinal preference ordering, the decision maker can hope for no more than a ranking. The cardinal differences between the *EV*s does not increase the decision maker's ability to rank alternatives and will probably be misinterpreted by the decision maker. If the decision maker desperately wants to know how much one alternative is preferred to another, he or she will be greatly tempted to assume that this desired information is provided by the relative magnitudes of the two alternative *EV*s. Representing this viewpoint, Samuelson brought our integrity into question when, after discussing whether consumer's surplus measures are useful for answering certain welfare questions, he stated "...all of these questions can be more conveniently (and more honestly!) answered in terms of the consumer's ordinal preference field..." (1947, p. 187).

Advocates of the practical view would probably accept the theoretical view if they felt we lived in a world where all aspects of the alternatives were considered when alternatives were ranked, and decisions were made solely on the basis of economic criteria. However, according to the practical view, money measures are quite useful in the real world. Bergson, for example, after summarizing the theoretical view, states "...I don't think I am saying anything that he [Samuelson] would consider very controversial when I hold that in practical work an ordinal ranking may sometimes do, but very often it is helpful, if not indispensible, to know not only more or less but how *much* more or less" (1975, pp. 40–41). Bergson goes on to argue that, in practice, when economists rank alternatives, they often do it on the basis of only a subset of the components of those alternatives: "...the evaluation [of welfare losses and gains] may be incomplete in diverse ways. It may well be incomplete even in respect to some costs and benefits of an economic sort. It would not be surprising, for example, if externalities such as pollution, congestion and the like were often omitted either wholly or in part" (1975, p. 41).

Bergson seems to suggest that although alternative states for an individual are identified in terms of prices, income level, taxes, fixed quantities of certain goods, characteristics, and risk, when economists rank projects, the ranking is done including only some of the components of each of the states (for example, prices).[16] The politician who must

[15]Even if one thinks that knowledge of the cardinal magnitudes of the *EV*s are of little use to policymakers, one would still probably find them personally interesting. Knowing another's *EV*s allows one to determine how much that person's *EV*s, if given to you, would increase your utility. Finding out someone else's *EV* for a project is like hearing that they just won a lottery; there is an automatic tendency to convert the dollar amount into utils on your scale.

[16]My 1982 paper has developed and estimated *EV*s that are a function of prices, characteristics of the goods (or activities), and characteristics of the individual.

choose one of the alternatives knows that the ranking might be wrong because of these omitted factors; therefore, in the words of Bergson,

> Before commiting himself, the public official will surely wish to know whether, according to any usual calculation, the measure in question would be economically advantageous or disadvantageous; but he might well want to ascertain also whether the gain or loss would be sufficient to make the action desirable or undesirable when due account is taken of costs and benefits beyond the reach of the calculations. True, once such costs and benefits are taken into account, an ordinal ranking is still all that is needed; but in arriving at such a ranking, the public official might be greatly aided if the calculated economic gain or loss were expressed in terms of some cardinally scaled metric. [1975, p. 41]

This argument suggests that the decision maker finds cardinally scaled measures of gains and losses meaningful because we (economists) do not do a very good job of including all the factors. The implication is that our economic ranking would be correct and there would be little reason for the cardinal measure if only we were clever enough to consider all the components of each alternative. Of course, in practice, decision makers do not rank solely on the basis of economic criteria. As Jagdish Bhagwati notes: "Policies are maintained or changed largely for noneconomic reasons; and the (economic) 'cost' involved is a magnitude that is commonly demanded and bandied about in discussions of public policy. Whether we like it or not, this is what the policy makers do want" (1965, p. 213). As Bergson implies, the politician needs cardinally scaled measures of the economic benefits of each alternative to compare them with the political costs of each alternative so as to determine his ordinal ranking of the projects.[17]

VII. A Concluding Example

Closing with an example that demonstrates many of my points, consider three government projects (A, B, C) that cause relative prices to change, and two individuals, Orville the ordinalist and Carleton the cardinalist. Assume we know only their demand equations and, by chance, both systems of demand equations are identical. Using the demand equations and applying McKenzie and Pearce's technique, we determine that the three EVs are $EV_A = \$100$, $EV_B = \$50$, and $EV_C = \$25$.[18]

What can we conclude? For both individuals, we know that project A is preferred to B which is preferred to C, but we don't know anything about how much Carleton prefers A over B or B over C. Orville doesn't think in terms of how much.

The EVs are meaningful in the cardinal sense that \$100 is twice as much money as \$50. Project A makes both individuals \$100 better off at current prices. Project B makes both individuals \$50 better off at current prices. Knowing these amounts and the level of current prices Carleton can figure out how much he prefers project A over B, but there is no way we are going to know unless he tells us. We can't say Carleton likes A twice as much as B.

It is obvious that the politician who must choose one of the three projects would rather know the EVs than just the rankings. Advocates of the practical view would say that the politician needs them so he can compare them to his implicit estimates of the political, and other omitted aspects of the projects. For example, maybe the politician knows that project A will cause an increase in precipitation. Neither Orville nor Carleton likes

[17]Cardinally scaled measures could also prove useful if the decision maker must make interpersonal utility comparisons (for example, see if a project has the potential for a Pareto improvement). Interpersonal utility comparisons are important for policy, but for the sake of clarity, this is a topic I have chosen not to discuss.

[18]Before continuing I should note that the McKenzie-Pearce technique is probably the easiest way to get the rankings if one does not get the demand equations by first specifying a preference ordering. However, if we got the demand equation by first assuming a preference ordering, we could have more easily used it to rank the projects and just reported to the policymaker that both individuals prefer A to B and B to C.

rain, but the economist did not consider this in ranking (his or her mistake). The politician then weighs the cardinal dollar advantage of A ($50) with the increased rain to try to determine whether Orville's and Carleton's rankings would change if they knew about the rain. The politician might also like to know the EVs because his ranking of the projects will depend on each of their political implications. The $50 advantage of A over B might not be sufficient to overcome the fact that A will decrease his chances of reelection.

Advocates of the theoretical view will be personally interested in knowing Orville's and Carleton's EVs, but will worry that the politician might misinterpret them and conclude that both Orville and Carleton like A twice as much as B.

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